Tree exercises

- **Build a tree**
  - People standing up are nodes that are currently in the tree
  - Point at a sitting down person to make them your child
  - Is it a binary tree? Is it a BST?
  - Traversals, height, deepest leaf?

- **How many different binary search trees are there with specified elements?**
  - E.g. given elements \(\{90, 13, 2, 3\}\), how many possible legal BSTs are there?
Searching, Maps, Tries (hashing)

- **Searching is a fundamentally important operation**
  - We want to search quickly, very very quickly
  - Consider searching using Google, ACES, issues?
  - In general we want to search in a *collection* for a *key*

- **We've searched using trees and arrays**
  - Tree implementation was quick: \( O(\log n) \) worst/average?
  - Arrays: access is \( O(1) \), search is slower

- **If we compare keys, \( \log n \) is best for searching \( n \) elements**
  - Lower bound is \( \Omega(\log n) \), provable
  - Hashing is \( O(1) \) on average, not a contradiction, why?
  - Tries are \( O(1) \) worst-case!! (ignoring length of key)
From Google to Maps

- If we wanted to write a search engine we’d need to access lots of pages and keep lots of data
  - Given a word, on what pages does it appear?
  - This is a map of words->web pages

- In general a map associates a key with a value
  - Look up the key in the map, get the value
  - Google: key is word/words, value is list of web pages
  - Anagram: key is string, value is words that are anagrams

- Interface issues
  - Lookup a key, return boolean: in map or value: associated with the key (what if key not in map?)
  - Insert a key/value pair into the map
Interface at work: MapDemo.java

- Key is a string, Value is # occurrences
  - Interface in code below shows how Map class works

```java
while (scanner.hasNext()) {
    String s = (String) scanner.next();
    Counter c = (Counter) map.get(s);
    if (c != null)
        c.increment();
    else
        map.put(s, new Counter());
}
```

- What clues are there for prototype of map.get and map.put?
  - What if a key is not in map, what value returned?
  - What kind of objects can be put in a map?
Accessing values in a map (e.g., print)

- Access every key in the map, then get the corresponding value
  - Get an iterator of the set of keys: `keySet().iterator()`
  - For each key returned by this iterator call `map.get(key)`
    ...

- Get an iterator over (key,value) pairs, there's a nested class called `Map.Entry` that the iterator returns, accessing the key and the value separately is then possible
  - To see all the pairs use `entrySet().iterator()`
External Iterator

- The Iterator interface access elements
  - Source of iterator makes a difference: cast required?

```java
Iterator it = map.keySet().iterator();
while (it.hasNext()){  
  Object value = map.get(it.next());
}
Iterator it2 = map.entrySet().iterator();
while (it2.hasNext()){  
  Map.Entry me = (Map.Entry) it2.next();  
  Object value = me.getValue();
}
```
Hashing: Log (10^{100}) is a big number

- Comparison based searches are too slow for lots of data
  - How many comparisons needed for a billion elements?
  - What if one billion web-pages indexed?

- Hashing is a search method: average case \(O(1)\) search
  - Worst case is very bad, but in practice hashing is good
  - Associate a number with every key, use the number to store the key
    - Like catalog in library, given book title, find the book

- A hash function generates the number from the key
  - Goal: Efficient to calculate
  - Goal: Distributes keys evenly in hash table
There will be collisions, two keys will hash to the same value
- We must handle collisions, still have efficient search
- What about birthday “paradox”: using birthday as hash function, will there be collisions in a room of 25 people?

Several ways to handle collisions, in general array/vector used
- Linear probing, look in next spot if not found
  - Hash to index $h$, try $h+1$, $h+2$, ..., wrap at end
  - Clustering problems, deletion problems, growing problems
- Quadratic probing
  - Hash to index $h$, try $h+1^2$, $h+2^2$, $h+3^2$, ..., wrap at end
  - Fewer clustering problems
- Double hashing
  - Hash to index $h$, with another hash function to $j$
  - Try $h$, $h+j$, $h+2j$, ...
Chaining with hashing

- **With n buckets each bucket stores linked list**
  - Compute hash value h, look up key in linked list table[h]
  - Hopefully linked lists are short, searching is fast
  - Unsuccessful searches often faster than successful
    - Empty linked lists searched more quickly than non-empty
  - Potential problems?

- **Hash table details**
  - Size of hash table should be a prime number
  - Keep load factor small: number of keys/size of table
  - On average, with reasonable load factor, search is $O(1)$
  - What if load factor gets too high? Rehash or other method
# Hashing problems

- **Linear probing, hash(x) = x, (mod tablesize)**
  - Insert 24, 12, 45, 14, delete 24, insert 23 (where?)

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- **Same numbers, use quadratic probing (clustering better?)**

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- **What about chaining, what happens?**
What about hash functions

- Hashing often done on strings, consider two alternatives

```java
public static int hash(String s)
{
    int k, total = 0;
    for(k=0; k < s.length(); k++){
        total += s.charAt(k);
    }
    return total;
}
```

- Consider `total += (k+1)*s.charAt(k)`, why might this be better?
  - Other functions used, *always mod result by table size*

- What about hashing other objects?
  - Need conversion of key to index, not always simple
  - Every object contains `hashCode()`!
A trie (from retrieval, but pronounced “try”) supports

- Insertion: put string into trie (delete and look up)
- These operations are $O(\text{size of string})$ regardless of how many strings are stored in the trie! Guaranteed!

In some ways a trie is like a 128 (or 26 or alphabet-size) tree, one branch/edge for each character/letter

- Node stores branches to other nodes
- Node stores whether it ends the string from root to it

Extremely useful in DNA/string processing

- Very useful for matching suffixes: suffix tree
Trie picture and code (see Trie.java)

- **To add string**
  - Start at root, for each char create node as needed, go down tree, mark last node

- **To find string**
  - Start at root, follow links
    - If null, not found
  - Check word flag at end

- **To print all nodes**
  - Visit every node, build string as nodes traversed

- **What about union and intersection?**

  Indicates word ends here