Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines

1. Given Turing Machines M1 and M2
   Notation for
   • Run M1
   • Run M2

2. Given Turing Machines M1 and M2
   Notation for
   • Run M1
   • If x is current symbol
     – then Run M2
3. Given Turing Machines M1, M2, and M3

Notation for

- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

M1
\[ S \rightarrow H \]

M2
\[ S' \rightarrow H' \]

M3
\[ S'' \rightarrow H'' \]

More Notation for Simplifying Turing Machines

Suppose \( \Gamma=\{a,b,c,B\} \)

- x is any element in \( \Gamma \)
- y is any element except x from \( \Gamma \)
- z is any element from \( \Gamma \)

1. s - start
2. R - move right
3. L - move left

4. x - write x (and don’t move)

5. Ra - move right until you see an a

6. La - move left until you see an a

7. R¬a - move right until you see anything that is not an a

8. L¬a - move left until you see anything that is not an a

9. h - halt in a final state

10. \( a, b \rightarrow w \)

   If the current symbol is a or b, let w represent the current symbol.
**Example**

Assume input string \( w \in \Sigma^+ \), \( \Sigma = \{a, b\} \).

If \( |w| \) is odd, then write a \( b \) at the end of the string. The tape head should finish pointing at the leftmost symbol of \( w \).

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example
Assume input string \( w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0 \)
For each \( a \) in the string, append a \( b \) to the end of the string.
input: \( abbabb \), output: \( abbabbbb \)
The tape head should finish pointing at the leftmost symbol of \( w \).

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.
Definition: An \textit{algorithm} for a function \( f: D \to R \) is a TM \( M \), which given input \( d \in D \), halts with answer \( f(d) \in R \).
Example: \( f(x + y) = x + y, x \) and \( y \) unary numbers.

\[
\begin{align*}
\text{start with:} & \quad 111+1111 \\
\uparrow & \\
\text{end with:} & \quad 1111111 \\
\uparrow &
\end{align*}
\]
**Example:** Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

Start with: \( abac \)

End with: \( abac0abac \)

**Algorithm:**

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol

\[ \text{s R 0 L} \quad \text{R} \quad \text{a,b,c} \quad \text{w} \quad \text{B R w L w} \]
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

\[
\begin{align*}
\text{start with:} & \quad \text{aaBbabca} \\
\text{end with:} & \quad \text{aaBBbaca}
\end{align*}
\]

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
**Example:** Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

- **start with:** babcaBba
- **end with:** bacaBBba

(similar to $S_R$)
**Example:** Add unary numbers

This time use shift.

**Example:** Multiply two unary numbers, \( f(x*y) = x*y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111*11 \\
& \uparrow \\
\text{end with:} & \quad 11111111 \\
& \uparrow 
\end{align*}
\]