Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:
Turing Machine (TM)

• invented by Alan M. Turing (1936)
• computational model to study algorithms

Definition of TM

• Storage
  – tape

• actions
  – write symbol
  – read symbol
  – move left (L) or right (R)

• computation
  – initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  – processing computation
    * move tape head left or right
    * read from and write to tape
  – computation halts
    * final state

Formal Definition of TM

A TM $M$ is defined by $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ where

• $Q$ is finite set of states
• $\Sigma$ is input alphabet
• $\Gamma$ is tape alphabet
• $B \in \Gamma$ is blank
• $q_0$ is start state
• $F$ is set of final states
• $\delta$ is transition function

$\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an 'a', then move into state $p$, write a 'b' on the tape and move to the right”.
TM as Language recognizer

**Definition:** Configuration is denoted by $\vdash$.

If $\delta(q,a) = (p,b,R)$ then a move is denoted

\[
abaqabba \vdash ababpbba
\]

**Definition:** Let $M$ be a TM, $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$. $L(M) = \{w \in \Sigma^*|q_0w \vdash x_1q_fx_2 \text{ for some } q_f \in F, x_1,x_2 \in \Gamma^*\}$

TM as language acceptor

$M$ is a TM, $w$ is in $\Sigma^*$,

- if $w \in L(M)$ then $M$ halts in final state
- if $w \notin L(M)$ then either
  - $M$ halts in non-final state
  - $M$ doesn’t halt

**Example**

$\Sigma = \{a, b\}$

Replace every second ’a’ by a ’b’ if string is even length.

- Algorithm
Example:

$L = \{a^n b^n c^n | n \geq 1\}$

Is the following TM Correct?

```
2;2,R
a;a,R

3;3,R
b;b,R

1;1,R

b;2,R
```

TM as a transducer

TM can implement a function: $f(w) = w'$

start with: $w$

end with: $w'$
**Definition:** A function with domain D is *Turing-computable* or *computable* if there exists TM
M=\( (Q, \Sigma, \Gamma, \delta, q_0, B, F) \) such that

\[
q_0 w \xrightarrow{\ast} q_f w
\]

\( q_f \in F \), for all \( w \in D \).

**Example:**

\( f(x) = 2x \)

x is a unary number

start with: \( \overline{111} \)

\[ \uparrow \]

end with: \( \overline{111111} \)

\[ \uparrow \]

Is the following TM correct?
Example:

$L = \{ww \mid w \in \Sigma^+ \}, \Sigma = \{a, b\}$