

## Your Camera as a Matrix

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## Coordinates

- Previous lecture assumed origin of coordinate system was middle of the lens
- Image plane:
  - Shifted back from the lens
  - Has different origin in X-Y
  - May be rotated (from nominal position)
  - Probably isn't sheered

## Linear Transformations

- Any 3x3 matrix operations is a linear transformation on points in (x,y,z)

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = A \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

- Which are physically possible/reasonable?

## Rotation in X-Y

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

## Scaling

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

## Sheer in X-Y

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} 1 & c & 0 \\ d & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

## What's Missing?

- Linear operations do not include translation
- Translation requires matrix multiplication followed by addition

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = A \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix}$$

- Such operations are *affine*

## Homogeneous Coordinates

- Affine transformations are linear transformations in a different space
- Trick for making affine operations linear:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = A \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

$$\begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & t_x \\ a_{21} & a_{22} & a_{23} & t_y \\ a_{31} & a_{32} & a_{33} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{pmatrix} A & t \\ 0' & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

## Ingredients for a Simple Camera Model

- Static parameters:
  - Rotation of image plane
  - Offset of image plane from lens ( $z'$ )
  - X-Y offset of image coordinates from lens center
- Dynamic parameter:
  - Magnification is a function of image distance

## Problems with Simple Model

- Lens Issues:
  - Spherical aberrations
    - Rays striking edges of lens not focused on exactly the same point as rays striking center
  - Coma
    - Similar to spherical aberration but caused by off-axis rays
  - Chromatic aberration
    - Different wavelengths refract slightly different amounts
  - Distortion (barrel, pincushion)
    - Magnification changes with distance from center
- Image plane may be out of calibration with device
  - Intrinsic vs. extrinsic parameters