Color Spaces

Ron Parr

CPS 1/296

Representing Color

- What is necessary to represent information needed to reconstruct a color scene?
- Map perceptual colors to wavelengths?
  - Leaves out brightness
  - Leaves out saturation
  - Is this even sufficient to cover all colors?
- Seems like we need 3 numbers
- Why not use stimulation levels for our cones?

CIE

- Commission Internationale de L'Éclairage
- Classic experiments measured cone sensitivity (up to linear transformation) using psychophysical experiments in 1931
- Revised slightly over time, but largely accurate – validated with biological/chemical experiments decades later
- Original CIE experiments used 3 monochromatic light sources
- http://upload.wikimedia.org/wikipedia/commons/2/22/CIExy1931_CIERGB.png

CIE Experiments: Things to Notice I

- Total energy was kept constant
  - Only two dimensions needed for graph
- Colors inside triangle could be produced by any convex combination of primary illuminants
  - Generalization: Convex hull of k primary colors circumscribes physically possible colors produced with k primaries
- How did they get colors outside of the triangle?
  - Is negative light possible?
  - Achieved by subtracting light from reference image

CIE Experiments: Things to Notice II

- Center point corresponds to white R=G=B
- Distance from center corresponds to saturation
- Angle around center corresponds to hue
  - Points along perimeter can be mapped to wavelengths
  - Shape suggests that no 3 pure colors are sufficient to cover entire gamut of viewable colors

CIE XYZ

- CIE proposed alternate set of primaries: XYZ
- Desiderata:
  - All colors representable as positive combination of XYZ
  - Y chosen to correspond to perceived brightness
- Consequences:
  - XYZ not directly physically realizable
  - Nevertheless, XYZ become the lingua franca of color science

Specifying Color Spaces

- Suppose we want to specify how a display (RGB) device works
- Specify the output primaries as a function of X,Y,Z components
  - $R = (R_x, R_y, R_z)$
  - $G = (G_x, G_y, G_z)$
  - $B = (B_x, B_y, B_z)$
- Q: What color is some combination of outputs from the RGB device?

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} R_x & G_x & B_x \\ R_y & G_y & B_y \\ R_z & G_z & B_z \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

Chromaticities

- Colors sometimes specified in terms of chromaticities
  - Lower case vs. Upper case!
  - r=R/(R+G+B)
  - g=G/(R+G+B)
  - b=B/(R+G+B)
- Good news: Scale invariance, 2 numbers
- Bad news: Scale invariance, 2 numbers
- Going from chromaticities to conversion matrix
  - Requires 4 sets of points
  - Linear system of 8 equations and 8 unknowns

Converting Between Color Spaces

- What color does R^1, G^1, B^1 on your LCD monitor correspond to on your plasma TV?
- NB: In principle, your computer software/firmware and your TV firmware should spare you (as a consumer) from ever worrying about this problem
- Suppose:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} R^1{}_x & G^1{}_x & B^1{}_x \\ R^1{}_y & G^1{}_y & B^1{}_y \\ R^1{}_z & G^1{}_z & B^1{}_z \end{pmatrix} \begin{pmatrix} R^1 \\ G^1 \\ B^1 \end{pmatrix} \qquad \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} R^2{}_x & G^2{}_x & B^2{}_x \\ R^2{}_y & G^2{}_y & B^2{}_y \\ R^2{}_z & G^2{}_z & B^2{}_z \end{pmatrix} \begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix}$$

Conversion continued

- Suppose:

$$C^1 = \begin{pmatrix} R^1{}_x & G^1{}_x & B^1{}_x \\ R^1{}_y & G^1{}_y & B^1{}_y \\ R^1{}_z & G^1{}_z & B^1{}_z \end{pmatrix} \qquad C^2 = \begin{pmatrix} R^2{}_x & G^2{}_x & B^2{}_x \\ R^2{}_y & G^2{}_y & B^2{}_y \\ R^2{}_z & G^2{}_z & B^2{}_z \end{pmatrix}$$

- Have: Way of going from RGB to XYZ
- Need: Way of going from XYZ back to RGB

Conversion continued

- Given:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = C^2 \begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix}$$

- Intuitively:

$$\begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix} = (C^2)^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Linear Algebra Digression/Review

- What is a matrix-vector product?
- Vector components are a recipe for combining the columns of the matrix

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} R^2{}_x & G^2{}_x & B^2{}_x \\ R^2{}_y & G^2{}_y & B^2{}_y \\ R^2{}_z & G^2{}_z & B^2{}_z \end{pmatrix} \begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix}$$

Identity and Inverse

- Identity matrix:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Properties

$$AI = IA = A$$

Inverse

- Properties:

$$A^{-1}A = AA^{-1} = I$$

- Exists if:
  - A is square
  - Has linearly independent columns

Basis

- A basis is a minimal set of column vectors
- Span of a basis is the set of possible column vectors producible with linear combinations of basis vectors, i.e., matrix vector products
- Columns of identity matrix are simplest basis

Converting XYZ to $R^2G^2B^2$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = C^2 \begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix}$$

$$(C^2)^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = (C^2)^{-1} C^2 \begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix}$$

$$(C^2)^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = I \begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix}$$

$$(C^2)^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix}$$

$$\begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix} = (C^2)^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Converting $R^1G^1B^1$ to $R^2G^2B^2$

- Convert to XYZ
- Convert to $R^2G^2B^2$

$$\begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix} = (C^2)^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix} = (C^2)^{-1} C1 \begin{pmatrix} R^1 \\ G^1 \\ B^1 \end{pmatrix}$$

Understanding Linear Algebra with Color

- Think of XYZ as the basis
- Coordinates of primaries in XYZ form columns of conversion matrix

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} R^1{}_x & G^1{}_x & B^1{}_x \\ R^1{}_y & G^1{}_y & B^1{}_y \\ R^1{}_z & G^1{}_z & B^1{}_z \end{pmatrix} \begin{pmatrix} R^1 \\ G^1 \\ B^1 \end{pmatrix}$$

- Matrix vector product specifies color mixing in terms of the XYZ primaries
- Conversion matrix is a *change of basis* from RGB to XYZ
- Conversion $(C^2)^{-1}C^1$ is a change of basis from $R^1G^1B^1$ to $R^2G^2B^2$

Linear Dependence

- Suppose the matrix inverse does not exist

$$\begin{pmatrix} R^2 \\ G^2 \\ B^2 \end{pmatrix} = (C^2)^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

- What does this tell us?

Understanding Dependence in Color Space

- Suppose inverse doesn't exist:
- RGB primaries form linearly dependent columns
- Some primary expressible as a linear combination of others
- Colors expressible with combinations of primaries form line or point
- Not all colors in XYZ can be expressed as combination of colors in RGB (even if we allow negative colors)

Difference Color Spaces

- Spaces for additive color devices are often RGB
- Sometimes more useful to express colors in terms of
  - Brightness (as a linear function of RGB) and differences
  - Y-R
  - Y-B
- Many variations on this…

Why use Y** spaces?

- Convenient for adding color to B&W TV
- Useful for compression

Polar Coordinates

- Polar coordinate representations catpure
  - Saturation
  - Brightness
  - Hue
- Some problems:
  - Some confusion about meaning of brightness
  - Awkward mathematically

Some Puzzles

- Suppose you tell your computer that an image is in a different colorspace. Should the image appearance change?
- Suppose you ask your computer to convert and image from one colorspace to another. Should the image appearance change?
- Suppose your camera stores images in sRGB (standard colorspace). Will converting to another colorspace let you see a wider range of colors?