# Filtering Tricks and Extensions

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## What we know so far

- Kalman filter
  - Assumes linear-Gaussian model
  - Exact, closed form solution given assumptions
- Particle filter
  - No modeling assumptions
  - Uses particles (simulation) to model distribution
  - May require many particles in some cases (= slow)

## What if dynamics aren't linear?

- Kalman filter assumes:
  - $x_{t+1} = f(x_t)$ + noise
  - f assumed linear
- For non-linear f
  - $x_{t+1} \sim x_t + f'(x_t)\Delta_{xt}$ + noise
  - First order linear approximation

## Making it work

- Kalman filter linear update: Ax
- Extended kalman filter update: $A'\Delta x$
- A' is the jacobian of the (non-linear) transition dynamics

$$A' = \begin{pmatrix} \frac{\partial x^1_{t+1}}{\partial x^1_t} & \cdots & \frac{\partial x^1_{t+1}}{\partial x^n_t} \\ \vdots & \ddots & \vdots \\ \frac{\partial x^n_{t+1}}{\partial x^1_t} & \cdots & \frac{\partial x^n_{t+1}}{\partial x^n_t} \end{pmatrix}$$

## Making it work II

- Must recompute Jacobian at every time step
- Derivatives taken at current mean
- $\Delta x$ must also be computed dynamically
- May also need to linearize observation model

## EKF Pros and Cons

- Pros:
  - Works well for small step sizes
  - Computationally tractable in most cases
  - Distribution remains bounded size
- Cons:
  - Gets into trouble for large step sizes
  - Requires real time differentiation

## Unscented Filter

- Idea:
  - Still represent distribution as a Gaussian
  - Use simulation to estimate parameters of Gaussian
- Implementation:
  - Pick a set of points with sample mean and sample covariance equal to current Gaussian (e.g. points evenly space along a 1 SD contour)
  - Push these points through the particle filter equations
  - Compute a new sample mean and covariance for the weighted, post-simulation points

## Surprising Facts about the UKF

- It stinks less than the EKF ☺
- Mean and covariance at least as good as (actually better than) EKF - accurate up to second order
- No real time differentiation
- Fast and easy to implement

## Why the UKF isn't a silver bullet

- Still assumes a Gaussian representation
- Can lead to (increasingly) bad approximations if distribution is not Gaussian
- …though probably no worse than EKF

## Rao Blackwellised Particle Filter

- Idea:
  - Sometimes it is possible to do parts of the filtering equations exactly, but not all
  - Sample the hard parts (particle filter)
  - Do the easy parts exactly (Kalman filter, or other method)
- Example application
  - Switching Kalman filter
  - Discrete variable selects from a set of possible linear dynamics at each time step
  - Mixture Kalman filter
    - Sample discrete variables
    - Do exact KF computations for continuous variables

## Mixture Kalman Filter Implementation

- Distribution is a set of particles, where each particle is a Gaussian distribution
- Can combine with EKF, UKF, etc.
- Efficiency, accuracy depend upon
  - Number of particles
  - Size of sampled state space
  - Concentration of particles
  - Approximation quality (EKF, UKF)

## Summary

- Kalman filter
  - Linear/Gaussian model
  - Closed form solution
  - Exact when assumptions met
- Extended Kalman Filter
  - Linearized Gaussian model
  - Closed form solution
  - Gaussian approximation to non-Gaussian reality
- Unscented Filter
  - Arbitrary model (almost)
  - Sample based approximation (for careful selection of sample points)
  - Fits Gaussian to samples
- Particle Filter
  - Arbitrary model (almost)
  - Sample based approximation
  - Quality depends upon #of particles, distribution
- Rao-Blackwellized Particle filter
  - Arbitrary model (almost)
  - Combines sampling w/closed form solution
  - Mixture of distributions approximation improves on particle filter accuracy