Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines.

1. Given Turing Machines $M_1$ and $M_2$
   
   Notation for
   - Run $M_1$
   - Run $M_2$

   
   \[ z \rightarrow z \rightarrow M_1 \rightarrow M_2 \]

   
   $z$ represents any symbol in $\Gamma$

2. Given Turing Machines $M_1$ and $M_2$
   
   Notation for
   - Run $M_1$
   - If $x$ is current symbol
     - then Run $M_2$

   
   \[ z \rightarrow z \rightarrow M_1 \rightarrow M_2 \]

   
   $z$ represents any symbol in $\Gamma$

   $x$ is an element of $\Gamma$
3. Given Turing Machines M1, M2, and M3

Notation for

- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

More Notation for Simplifying Turing Machines

Suppose $\Gamma=\{a,b,c,B\}$

- z is any symbol in $\Gamma$
- x is a specific symbol from $\Gamma$

1. s - start
2. R - move right
3. L - move left

4. x - write x (and don’t move)

5. Rₐ - move right until you see an a

6. Lₐ - move left until you see an a

7. Rₐa - move right until you see anything that is not an a

8. Lₐa - move left until you see anything that is not an a

9. h - halt in a final state

10. \( \frac{a,b}{w} \)
    
    If the current symbol is a or b, let w represent the current symbol.
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example
Assume input string \( w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0 \)

For each \( a \) in the string, append a \( b \) to the end of the string.

input: \( abbabb \), output: \( abbabbbb \)

The tape head should finish pointing at the leftmost symbol of \( w \).

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function \( f:D \rightarrow \mathbb{R} \) is a TM \( M \), which given input \( d \in D \), halts with answer \( f(d) \in \mathbb{R} \).

Example: \( f(x + y) = x + y \), \( x \) and \( y \) unary numbers.

\[
\begin{align*}
\text{start with:} & \quad 111+1111 \\
& \uparrow \\
\text{end with:} & \quad 1111111 \\
& \uparrow
\end{align*}
\]
**Example:** Copy a String, \( f(w) = w0w, w \in \Sigma^*, \Sigma = \{a, b, c\} \)

Denoted by \( C \)

| Start with: \( \text{abac} \) | \( \uparrow \) |
| End with: \( \text{abac0abac} \) | \( \uparrow \) |

**Algorithm:**

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol

\[ \begin{array}{c}
\text{s R 0 L}
\end{array} \quad \begin{array}{c}
\text{a,b,c}
\end{array} \quad \begin{array}{c}
w
\end{array} \quad \begin{array}{c}
\text{R}
\end{array} \quad \begin{array}{c}
\text{B}
\end{array} \quad \begin{array}{c}
w
\end{array} \quad \begin{array}{c}
\text{R}
\end{array} \quad \begin{array}{c}
\text{B}
\end{array} \quad \begin{array}{c}
w
\end{array} \quad \begin{array}{c}
\text{L}
\end{array} \quad \begin{array}{c}
\text{B}
\end{array} \quad \begin{array}{c}
\text{R}
\end{array} \quad \begin{array}{c}
h
\end{array} \quad \begin{array}{c}
\text{B}
\end{array} \]
**Example:** Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

\[
\begin{align*}
\text{start with: } & \quad \text{aaBbabca} \\
\text{end with: } & \quad \text{aaBBbaca}
\end{align*}
\]

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: babcaBba

end with: bacaBBba

(similar to $S_R$)
Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, \( f(x*y) = x*y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111 \uparrow 11 \\
& \quad \uparrow \\
\text{end with:} & \quad 1111111 \\
& \quad \uparrow
\end{align*}
\]