What will we do in CPS 140?

Questions

- Can you write a program to determine if a string is an integer?
  
  9998.89 8abab 789342

- Can you do this if your machine had no additional memory other than the program? (can’t store any values and look at them again)

- Can you write a program to determine if the following are arithmetic expressions?
  
  $$((34 + 7 * (18/6)))$$

  $$((((((a + b) + c) * d(e + f))))))$$

- Can you do this if your machine had no additional memory other than the program?

- Can you write a program to determine the value of the following expression?
  
  $$((34 + 7 * (18/6)))$$

- Can you write a program to determine if a file is a valid C++/Java program?

- Can you write a program to determine if a Java/C++ program given as input will ever halt?

Language Hierarchy

![Language Hierarchy Diagram]

Grammars

- unrestricted grammar
- CFG
- regular grammar

Automata

- Turing machine
- pushdown automata
- finite automata
Power of Machines

<table>
<thead>
<tr>
<th>automata</th>
<th>Can do?</th>
<th>Can’t do?</th>
</tr>
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<tbody>
<tr>
<td>finite automata (FA)</td>
<td>integers</td>
<td>arith expr</td>
</tr>
<tr>
<td>(no memory)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pushdown automata (PDA)</td>
<td>arith expr</td>
<td>compute expr</td>
</tr>
<tr>
<td>(only memory is stack)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turing machines (TM)</td>
<td>compute expr</td>
<td>decide if halts</td>
</tr>
<tr>
<td>(infinite memory)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Application

Compiler

- Question: Java/C++ program - is it valid?
- Question: language L, program P - is P valid?

Stages of a Compiler

```
C++ program

 lexical analysis

 tokens

 syntax analysis

 parse tree

 code generation

 assembly language program
```
Set Theory - Read Chapter 1 Linz.

A Set is a collection of elements.

$A = \{1, 4, 6, 8\}$, $B = \{2, 4, 8\}$, $C = \{3, 6, 9, 12, \ldots\}$, $D = \{4, 8, 12, 16, \ldots\}$

- (union) $A \cup B = \{1, 2, 4, 6, 8\}$
- (intersection) $A \cap B = \{4\}$
- $C \cap D = \{6, 12, 18, \ldots\}$
- (member of) $42 \in C$?
- (subset) $B \subseteq C$?
- $B \cap A \subseteq D$?
- (product) $A \times B = \{(a, b) \mid a \in A, b \in B\}$
- $|B| =$
- $|A \times B| =$
- $\emptyset \in B \cap C$?
- (powerset) $2^B = \{\emptyset, B, C, \ldots\}$

**Example** What are all the subsets of $\{3, 5\}$?

How many subsets does a set $S$ have?

| $|S|$  | number of subsets |
|------|-------------------|
| 0    | 1                 |
| 1    | 2                 |
| 2    | 4                 |
| 3    | 8                 |
| 4    | 16                |

How do you prove? Set $S$ has $2^{|S|}$ subsets.
Technique: Proof by Induction

1. Basis: P(1)? Prove smallest instance is true.
2. Induction Hypothesis - I.H.
   Assume P(n) is true for 1,2,...,n
3. Induction Step - I.S.
   Show P(n+1) is true (using I.H.)

Proof of Example:

1. Basis:
2. I.H. Assume
3. I.S. Show

Ch. 1: 3 Major Concepts

- languages
- grammars
- automata

Languages

- \( \Sigma \) - set of symbols, alphabet
- string - finite sequence of symbols
- language - set of strings defined over \( \Sigma \)

Examples

- \( \Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \)
  \( L = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, \ldots\} \)
- \( \Sigma = \{a, b, c\} \)
  \( L = \{ab, ac, cabb\} \)
- \( \Sigma = \{a, b\} \)
  \( L = \{a^n b^n \mid n > 0\} \)
Notation

- symbols in alphabet: a, b, c, d, ...
- string names: u,v,w,...

Definition of concatenation

Let \( w = a_1a_2\ldots a_n \) and \( v = b_1b_2\ldots b_m \)

Then \( w \circ v \) OR \( wv = \)

See book for formal definitions of other operations.

String Operations

strings: \( w = abbc \), \( v = ab \), \( u = c \)

- size of string
  \( |w| + |v| = \)
- concatenation
  \( v^3 = vvv = vovov = \)
- \( v^0 = \)
- \( u^R = \)
- \( |vv^Rw| = \)
- \( ab \circ \lambda = \)

Definition

\( \Sigma^* = \) set of strings obtained by concatenating 0 or more symbols from \( \Sigma \)

Example

\( \Sigma = \{a, b\} \)

\( \Sigma^* = \)

\( \Sigma^+ = \)

Examples

\( \Sigma = \{a, b, c\}, L_1 = \{ab, bc, aba\}, L_2 = \{c, bc, bcc\} \)

- \( L_1 \cup L_2 = \)
- \( L_1 \cap L_2 = \)
- \( L_1^c = \)
- \( \overline{L_1 \cap L_2} = \)
- \( L_1 \circ L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\} = \)
Definition

\[ L^0 = \{ \lambda \} \]
\[ L^2 = L \circ L \]
\[ L^3 = L \circ L \circ L \]
\[ L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \ldots \]
\[ L^+ = L^1 \cup L^2 \cup L^3 \ldots \]

Grammars

grammar for english

\[ <\text{sentence}> \rightarrow <\text{subject}><\text{verb}><\text{d.o.}> \]
\[ <\text{subject}> \rightarrow <\text{noun}> | <\text{article}><\text{noun}> \]
\[ <\text{verb}> \rightarrow \text{hit} | \text{ran} | \text{ate} \]
\[ <\text{d.o.}> \rightarrow <\text{article}><\text{noun}> | <\text{noun}> \]
\[ <\text{noun}> \rightarrow \text{Fritz} | \text{ball} \]
\[ <\text{article}> \rightarrow \text{the} | \text{an} | \text{a} \]

Examples (derive a sentence)

Fritz hit the ball.

\[ <\text{sentence}> \rightarrow <\text{subject}><\text{verb}><\text{d.o.}> \]
\[ \rightarrow <\text{noun}><\text{verb}><\text{d.o.}> \]
\[ \rightarrow \text{Fritz} <\text{verb}><\text{d.o.}> \]
\[ \rightarrow \text{Fritz hit} <\text{d.o.}> \]
\[ \rightarrow \text{Fritz hit} <\text{article}><\text{noun}> \]
\[ \rightarrow \text{Fritz hit the} <\text{noun}> \]
\[ \rightarrow \text{Fritz hit the} \text{ ball} \]

Can we also derive the sentences?

The ball hit Fritz.

The ball ate the ball

Syntactically correct?

Semantically correct?
Grammar

$$G=(V,T,S,P)$$ where

- \( V \) - variables (or nonterminals)
- \( T \) - terminals
- \( S \) - start variable (\( S \in V \))
- \( P \) - productions (rules)
  \( x \rightarrow y \) “means” replace \( x \) by \( y \)
  \( x \in (V \cup T)^+, \ y \in (V \cup T)^* \)
  where \( V, T, \) and \( P \) are finite sets.

Definition

\( w \Rightarrow z \) \quad w \text{ derives } z

\( w \Rightarrow z \) \quad \text{derives in } 0 \text{ or more steps}

\( w \Rightarrow z \) \quad \text{derives in } 1 \text{ or more steps}

Definition

\( G=(V,T,S,P) \)

\( L(G)=\{w\in T^* \mid S \Rightarrow w\} \)

Example

\( G=(\{S\}, \{a,b\}, S, P) \)

\( P=\{S\rightarrow aaS, S\rightarrow b\} \)

\( L(G)=\)

Example

\( L(G) = \{a^n c b^n \mid n > 0\} \)

\( G = \)

Automata Abstract model of a digital computer