Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:
Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape

- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)

- computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM $M$ is defined by $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ where

- $Q$ is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $B \in \Gamma$ is blank
- $q_0$ is start state
- $F$ is set of final states
- $\delta$ is transition function
  
  $\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an ’a’, then move into state $p$, write a ’b’ on the tape and move to the right”.

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TM as Language recognizer

Definition: Configuration is denoted by ⊢.

if δ(q,a) = (p,b,R) then a move is denoted

abaqabba ⊢ ababpbba

Definition: Let M be a TM, M=(Q,Σ,Γ,δ,q0,B,F). L(M) = \{ w ∈ Σ∗ | q0 w ⊢ x_1 q_f x_2 for some q_f ∈ F, x_1, x_2 ∈ Γ∗ \}

TM as language acceptor

M is a TM, w is in Σ∗,

- if w ∈ L(M) then M halts in final state
- if w ∉ L(M) then either
  - M halts in non-final state
  - M doesn’t halt

Example

Σ = \{a, b\}

Replace every second ‘a’ by a ‘b’ if string is even length.

- Algorithm
Example:

\[ L = \{ a^n b^n c^n | n \geq 1 \} \]

Is the following TM Correct?

\[ \text{TM as a transducer} \]

TM can implement a function: \( f(w) = w' \)

\[
\begin{align*}
\text{start with:} & \quad w \\
\text{end with:} & \quad w'
\end{align*}
\]
**Definition:** A function with domain $D$ is *Turing-computable* or *computable* if there exists TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ such that

$$q_0 w \stackrel{*}{\vdash} q_f f(w)$$

$q_f \in F$, for all $w \in D$.

**Example:**

$f(x) = 2x$

$x$ is a unary number

```
start with:       111
↑
end with:        111111
↑
```

Is the following TM correct?

![Diagram of TM](image_url)
Example:

$L = \{ww \mid w \in \Sigma^+ \}, \Sigma = \{a, b\}$