Questions may continue on the back. Please write clearly. What I cannot read, I will not grade. Typed homework is preferable. A good compromise is to type the words and write the math by hand.

1. Let

\[ A = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad C = [ -1, \ 2 ] . \]

Evaluate the following quantities by hand, showing intermediate operations. If an operation is undefined, so state.

(a) \( AB \)
(b) \( BA \)
(c) \( B^T A \)
(d) \( CA \)
(e) \( A^2 \)
(f) \( A^T A \)
(g) \( B^2 \)
(h) \( \|B\| \)

2. For this problem, it’s fine to use Matlab for the calculations. Let

\[ D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.05 \end{bmatrix}, \quad F = \begin{bmatrix} 0.8 & 0.3 \\ 0 & 0.8 \end{bmatrix}. \]

(a) Write the five powers \( D^n \) for \( n = 1, \ldots, 5 \).

(b) Write a closed-form\(^1\) formula for \( D^n \). It’s OK to “explain” the formula: “if \( X \) is the case, do \( Y \), otherwise do \( Z \)...”.

(c) Write the three powers \( E^n \) for \( n = 0, 1, 2 \).

(d) Write a general formula for

\[ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^n \quad \text{for} \quad n = 0, 1, \ldots \]

(e) Write Matlab code that plots the values of the two entries in the first row of the matrix \( F^n \) for \( n = 0, \ldots, 30 \) on the same diagram. Turn in your code and your diagram. Make sure axes are labeled. Use the Matlab \texttt{legend} command to show which plot is which.

3. Matlab code for the function \texttt{LDSDDS} is provided on the homework web page. This function computes the response of a linear, deterministic, stationary, discrete, dynamic system, as described in the supplementary notes. Write a Matlab script that uses \texttt{LDSDDS} to reproduce the eight diagrams in Figure 5, page 35 and Figure 6, page 36 of the class notes. Label the axes, use \texttt{legend} to show which plot is which, and use \texttt{title} to distinguish the four diagrams. There is no need to number the points on the plot. For each plot, 100 points are shown. Write clean code: no cutting and pasting! Hand in your code and your eight diagrams (printed on fewer than eight pages, if possible).

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\(^1\)As explained in class, such a formula is in closed form if the time needed for its computation does not grow with \( n \). So a recurrence is \textit{not} OK.