Questions may continue on the back. Please write clearly. What I cannot read, I will not grade. Typed homework is preferable. A good compromise is to type the words and write the math by hand.

1. Let

\[ f(a) = \begin{cases} 
  ca & \text{for } a < 1/2 \\
  c(1 - a) & \text{for } a \geq 1/2 
\end{cases} \]

where \( c \) is a positive, real constant. The graph of this function is shown in Figure 1.

![Figure 1 The graph of the function \( f(a) \).](image)

(a) Write a Matlab function with the following header:

```matlab
function a = d(c, a0, N)
```

that takes a positive, real number \( c \), a real number \( a0 \) between 0 and 1 (inclusive), and an integer \( N \), and returns a row vector \( a \) of the \( N \) values obtained by running the iteration

\[ a(n + 1) = f(a(n)) \]

\( N - 1 \) times, with initial value \( a(0) = a0 \), and where \( f \) is the function defined above. The value \( a0 \) should be the first entry of \( a \). Your code should check that \( c \) is positive and that \( a0 \) is between 0 and 1 (inclusive) (and issue an error message otherwise). No other argument checks are required. Show your code, and the plots obtained with each of the commands

```matlab
plot(d(0.8, 0.3, 30))
plot(d(1, 0.3, 30))
plot(d(1, 0.8, 30))
plot(d(1.2, 0.3, 30))
plot(d(1.5, 0.3, 30))
```

Also show a single plot with two graphs superimposed as follows:

```matlab
N = 50;
clf
plot(d(1.8, 0.6, N));
hold on
plot(d(1.8, 0.6001, N), ':k');
```
The command `clf` clears the figure, and `hold on` causes the second graph to be superimposed over the first. The string `'r'` passed to `plot` causes the second plot to be drawn with a black (k), dotted (:) line. Use the `title` or `legend` commands to show which plot is which.

(b) For each of the following three sets of values of \( c \),
\[
0 \leq c < 1 \quad , \quad c = 1 \quad , \quad c > 1
\]
find the fixed point or fixed points of the dynamic system
\[
a(n + 1) = f(a(n)).
\]

(c) Which fixed points are stable, and why?

2. Draw (by hand is fine, but be neat) the cobweb diagrams for a few iterations of the linear system
\[
a(n + 1) = (1 + r)a(n) + b
\]
\[
a(0) = a_0
\]
for the following values of \( r, b, \) and \( a_0 \)

(a) \( r = -0.5, b = 1, a_0 = 0.25 \).

(b) \( r = -0.5, b = 1, a_0 = 4 \).

(c) \( r = 1, b = -2, a_0 = 0.25 \).

(d) \( r = 1, b = -2, a_0 = 3 \).

In all diagrams, use a different color or line style for the identity function and for the function \( f(a) = (1 + r)a + b \). Also state for each case what the fixed point is, and whether it is stable or unstable.