CPS 140 - Mathematical Foundations of CS
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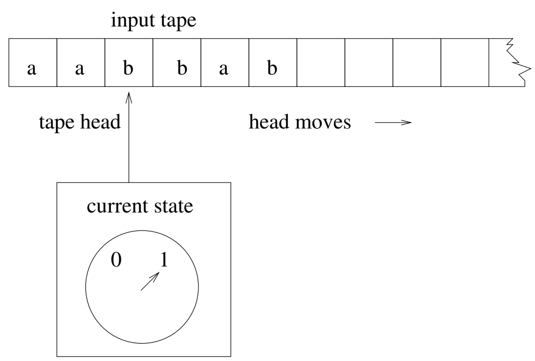
Section: Finite Automata (Ch. 2) (handout)

Deterministic Finite Accepter (or Automata)

A DFA=(Q,Σ,δ,q_0,F)



where

Q is finite set of states
Σ is tape (input) alphabet
q_0 is initial state
F $\subseteq$ Q is set of final states.
δ:Q$\times\Sigma\rightarrow$Q

Example: Create a DFA that accepts even binary numbers.

Transition Diagram:

M=(Q,Σ,δ,q_0,F) =

Tabular Format

	0	1
q0		
q1		

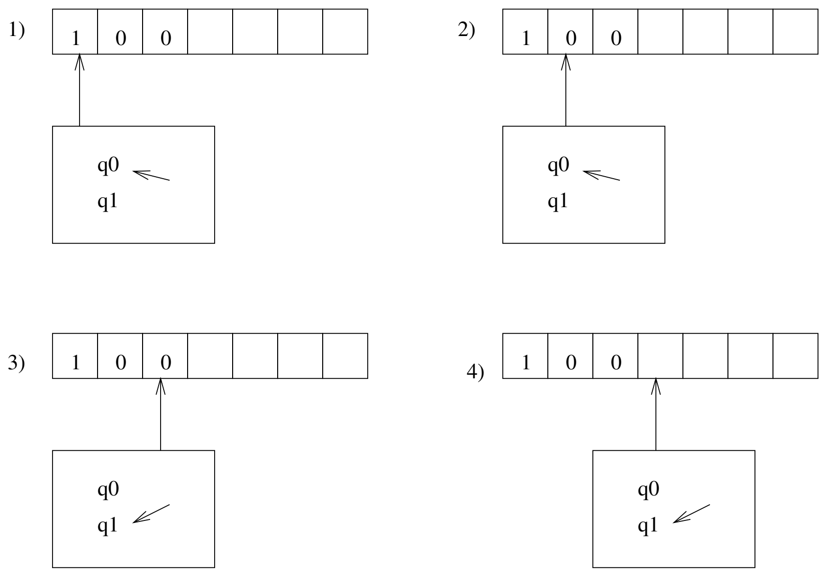
Example of a move: δ(q0,1)=

Algorithm for DFA:

```
Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
    q = δ(q,s)
    s = next symbol to the right on tape
if q∈F then accept
```

Example of a trace: 11010

Pictorial Example of a trace for 100:



Definition:

$\delta^*(q, \lambda) = q$

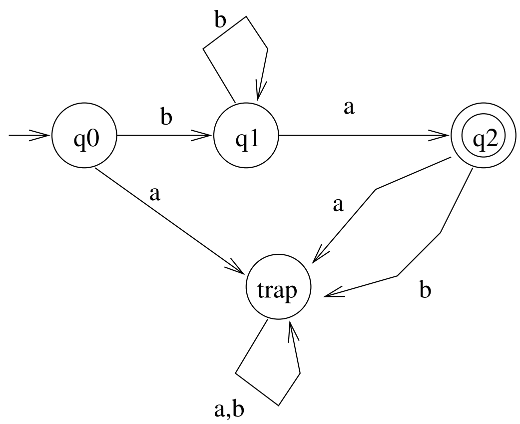
$\delta^*(q, wa) = \delta(\delta^*(q, w), a)$

Definition The language accepted by a DFA M=(Q,Σ,δ,q_0,F) is set of all strings on Σ accepted by M. Formally,

L(M)=$\{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\}$

Trap State

Example: L(M) = $\{b^n a \mid n > 0\}$



You don't need to show trap states! Any arc not shown will by default go to a trap state.

Example: Create a DFA that accepts even binary numbers that have an even number of 1's.

Example:

L = $\{w \in \Sigma^* \mid$ w has an even number of a's and an even number of b's$\}$

Definition A language is regular iff there exists DFA M s.t. L=L(M).

Chapter 2.2

Nondeterministic Finite Automata (or Accepter)

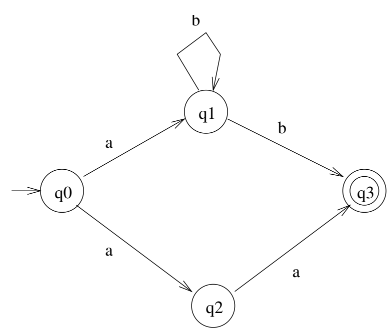
Definition

An NFA=(Q,Σ,δ,q_0,F)

where

Q is finite set of states
Σ is tape (input) alphabet
q_0 is initial state
F $\subseteq$ Q is set of final states.
δ:Q$\times$($\Sigma \cup \{\lambda\}$)$\rightarrow 2^Q$

Example



Note: In this example $\delta(q_0, a) =$

L=

Example

L=$\{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\}$

Definition $q_j \in \delta^*(q_i, w)$ if and only if there is a walk from q_i to q_j labeled w.

Example From previous example:

$\delta^*(q_0, ab)$=

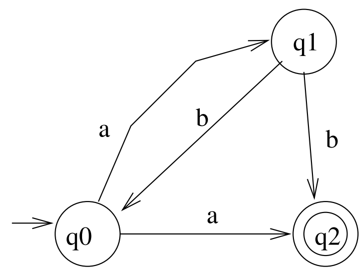
$\delta^*(q_0, aba)$=

Definition: For an NFA M, L(M)=$\{w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset\}$

The language accepted by nfa M is all strings w such that there exists a walk labeled w from the start state to final state.

2.3 NFA vs. DFA: Which is more powerful?

Example:



Theorem Given an NFA M_N=$(Q_N, \Sigma, \delta_N, q_0, F_N)$, then there exists a DFA M_D=$(Q_D, \Sigma, \delta_D, q_0, F_D)$ such that $L(M_N) = L(M_D)$.

Proof:

We need to define M_D based on M_N.

$Q_D =$

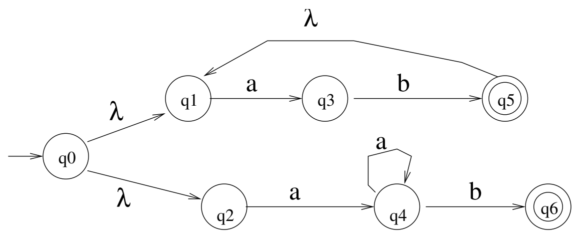
$F_D =$

δ_D :

Algorithm to construct M_D

1. start state is $\{q_0\} \cup$ closure(q_0)
2. While can add an edge
   (a) Choose a state A=$\{q_i, q_j, ...q_k\}$ with missing edge for $a \in \Sigma$
   (b) Compute B = $\delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \ldots \cup \delta^*(q_k, a)$
   (c) Add state B if it doesn't exist
   (d) add edge from A to B with label a
3. Identify final states
4. if $\lambda \in L(M_N)$ then make the start state final.

Example:



Minimizing Number of states in DFA

Why?

Algorithm

- Identify states that are indistinguishable

  These states form a new state

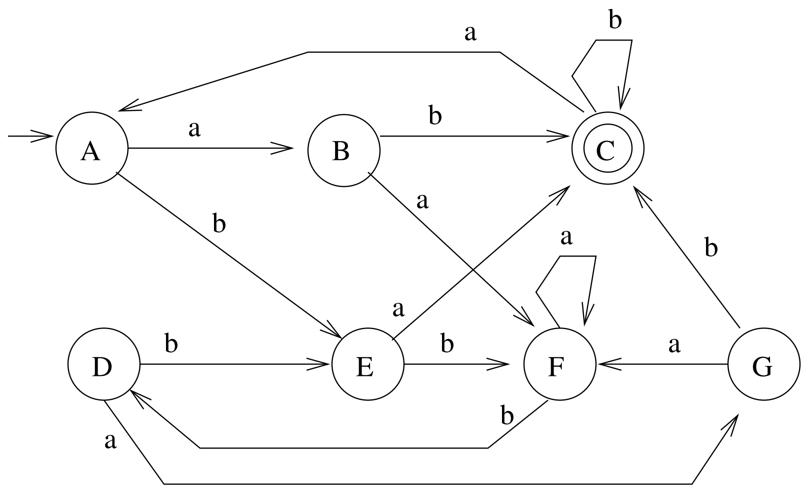
Definition Two states p and q are indistinquishable if for all $w \in \Sigma^*$

$$\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \in F$$
$$\delta^*(p, w) \notin F \Rightarrow \delta^*(q, w) \notin F$$

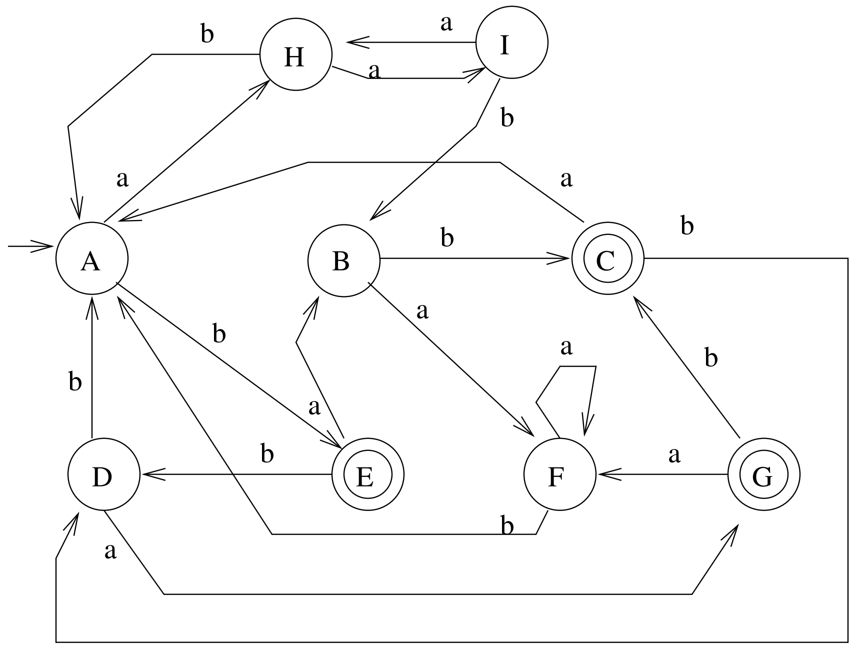
Definition Two states p and q are distinquishable if $\exists\ w \in \Sigma^*$ s.t.

$\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \notin F$ OR
$\delta^*(q, w) \notin F \Rightarrow \delta^*(p, w) \in F$

Example:



Example:



Properties and Proving - Problem 1

Consider the property Replace_one_a_with_b or R1awb for short. If L is a regular, prove R1awb(L) is regular.

The property R1awb applied to a language L replaces one a in each string with a b. If a string does not have an a, then the string is not in R1awb(L).

Properties and Proving - Problem 2

Consider the property Truncate_all_preceeding_b's or TruncPreb for short. If L is a regular, prove TruncPreb(L) is regular.

The property TruncPreb applied to a language L removes all preceeding b's in each string. If a string does not have an preceeding b, then the string is the same in TruncPreb(L).