

## Homework 1

- §1 Show that  $\Sigma_2^P = \mathbf{NP}^{\mathbf{SAT}}$ .
- §2 Show that  $\mathbf{SPACE}(n) \neq \mathbf{NP}$ . (Hint: Use padding, mentioned in the notes for Lecture 1.)
- §3 Can you give a definition of  $\mathbf{NEXPTIME}$  in terms of certificates as we did for  $\mathbf{NP}$ ? If not, report your best attempt.
- §4 Say that a class  $C_1$  is *superior* to a class  $C_2$  if there is a machine  $M_1$  in class  $C_1$  such that for every machine  $M_2$  in class  $C_2$  and every large enough  $n$ , there is an input of size between  $n$  and  $n^2$  on which  $M_1$  and  $M_2$  answer differently.
- (a) Is  $\mathbf{DTIME}(n^{1.1})$  superior to  $\mathbf{DTIME}(n)$ ?
  - (b) Is  $\mathbf{NTIME}(n^{1.1})$  superior to  $\mathbf{NTIME}(n)$ ?
- §5 Suppose we define the *logspace hierarchy* in analogy with the polynomial hierarchy using logspace machines that can use alternation. Does this hierarchy collapse by Immerman's theorem ( $\mathbf{NL} = \mathbf{coNL}$ )?

### 6. Prove that integer programming is NP-complete by using a reduction from 3-CNF-SAT

Given a set of  $m$  linear constraints over  $n$  variables, the integer-programming problem asks whether there is an *integer*  $n$ -vector  $x$  giving values for each of the  $n$  variables such that all the constraints are satisfied.

### 7. Show the subset sum problem is NP complete.

Given a set  $S$  of  $n$  integers and an integer  $K$ , the subset sum problem is:  
Is there a subset  $S'$  of  $S$  such that  $K$  equals the sum of the elements of the sets in  $S'$ .