

**CPS240**  
**Homework 3**  
**Due March 25 2009**

1 Recall the definition of language classes ZPP, RP and its complement coRP given in the reading.

**Show that  $ZPP = RP \cap \text{coRP}$ .**

2. Language class BP·NP is defined:  $\text{BP} \cdot \text{NP} = \{L : L \leq_p 3\text{SAT}\}$ .

A nondeterministic circuit has two inputs  $x, y$ . We say that it accepts  $x$  iff there exists  $y$  such that  $C(x, y) = 1$ . The *size* of the circuit is measured as a function of  $|x|$ .

Let  $\text{NP}/poly$  be the languages that are decided by polynomial size nondeterministic circuits.

**Show that  $\text{BP} \cdot \text{NP} \subseteq \text{NP}/poly$ .**

3. Language class BPL is defined:

$L \in \text{BPL}$  if there is a  $O(\log n)$ -space PTM  $M$  such that for every  $x \in \{0, 1\}^*$ ,  
 $\Pr[M(x) = L(x)] \geq 2/3$ .

**Show that  $\text{BPL} \subseteq \text{P}$**

Hint: Use dynamic programming (involving matrix products) to determine the probability the probabilistic machine ends up in the accept configuration.