Part I.

POLYNOMIALS
TRIGONOMETRIC POLYNOMIALS
PIECEWISE POLYNOMIALS

1. As discussed in the class, $P_m$ and $T_n$ are vector spaces, for $m, n \geq 0$.

   (a) describe a relationship between $P_n$ and $P_{n+1}$,
   (b) describe a relationship between $T_n$ and $T_{n+1}$,
   (c) discuss the relationships between $P_m$ and $T_n$ for any pair of $m$ and $n$.

2. Specify an orthogonal basis of $T_n$ and the recursive relationship among the basis functions.

3. Verify that \{1, x, \ldots, x^n\} form a basis for $P_n$. Describe an approach to getting an orthogonal basis from the natural one. Find a recursion among the orthogonal basis functions.

4. Describe the properties of the central B-spline functions $B_k$, $k \geq 0$, and the recursive evaluation of the functions.

5. Find the Fourier transform of the central B-spline functions and describe certain properties of the transformed functions.

6. Describe briefly the method via Taylor’s expansion for approximating a smooth function with piecewise polynomials.
Part II.

1. Experiment with the provided MATLAB implementations of the GAXPY operation; make observations and offer explanations of the observed phenomenons.

2. Provide a MATLAB function for the root extraction $z^4 = 4$ with Newton’s iteration. Specify the initialization scheme and termination criteria.

3. Provide a MATLAB script that calls the root extraction function and visualizes the iteration behavior over the convex region $|x + y| \leq 2$.

4. Describe your observation based on the experimental results.

Optional Provide a comparison with some other method for the root extraction.