Part I.
POLYNOMIALS
TRIGONOMETRIC POLYNOMIALS
PIECEWISE POLYNOMIALS

1. As discussed in the class, P_m and T_n are vector spaces, for $m, n \geq 0$.
   (a) describe a relationship between P_n and P_{n+1} ,
   (b) describe a relationship between T_n and T_{n+1} ,
   (c) discuss the relationships between P_m and T_n for any pair of m and n.
2. Specify an orthogonal basis of T_n and the recursive relationship among the basis functions.
3. Verify that $\{1, x, \cdots, x^n\}$ form a basis for P_n. Describe an approach to getting an orthogonal basis from the natural one. Find a recursion among the orthogonal basis functions.
4. Describe the properties of the central B-spline functions B_k, $k \geq 0$, and the recursive evaluation of the functions.
5. Find the Fourier transform of the central B-spline functions and describe certain properties of the transformed functions.
6. Describe briefly the method via Taylor's expansion for approximating a smooth function with piecewise polynomials.

Part II.

1. Experiment with the provided MATLAB implementations of the GAXPY operation; make observations and offer explanations of the observed phenomenons.

2. Provide a MATLAB function for the root extraction $z^4 = 4$ with Newton's iteration. Specify the initialization scheme and termination criteria.

3. Provide a MATLAB script that calls the root extraction function and visualizes the iteration behavior over the convex region $|x + y| \leq 2$.

4. Describe your observation based on the experimental results.

Optional Provide a comparison with some other method for the root extraction.