Interpolation and Regression Methods

Jan. 28 - Feb. 10

Part I

1. Extend the Newton’s interpolation approach to the following interpolation problem,

\[(x_i, y_i, y_i', b_i), \quad i = 0 : n;\]

where \(y_i'\) denotes the derivative value at \(x_i\) and \(b_i\) is a binary flag indicating whether or not the derivative interpolation condition is to be met.

2. Extend the LS linear regression approach to the following data fitting problem with data \((x_i, y_i), i = 0 : n,\) subject to the equality constraints,

\[p(x_j) = y_j, \quad j \in S_n\]

where \(S_n\) is a small subset of \(\{0, 1, \ldots, n\}\) and \(p\) is a polynomial interpolant.

3. Describe the difference in the update computation at each elimination step between the LU factorization with pivoting and the QR factorization of a matrix.

4. Find some common features and differences between the following two matrix operators

\[H_1 = I - uu^T, \quad H_2 = I - 2uu^T\]

where \(I\) is the identity matrix, \(u\) is a real vector with unit Euclidean length \(u^Tu = 1.\)

5. Describe the structures of the Laplacian matrices on a finite \(d\) dimensional grid, \(d = 1, 2, 3.\)
Part II

1. The Runge function is defined as follows.

\[ \text{runge}(x) = \frac{1}{1 + x^2}, \quad x \in (-\infty, \infty). \]

Compare the following interpolation methods on the interval \([-5, 5]\), as the number of interpolation nodes increases.

(a) the Vandermonde approach

(b) the Cubic spline approach

(c) (optional) the B-spline approach

Built-in or provided MATLAB functions or scripts:
\text{pval, ppval, runge, rungeView, pPolyEval}

2. Provide review comments on Problem 4 of Exercise Set 3.5. in the textbook (parametric curves).