Interpolation and Regression Methods

Jan. 28 - Feb. 10

Part I

1. Extend the Newton's interpolation approach to the following interpolation problem,
$$(x_i, y_i, y_i' b_i), \quad i = 0 : n;$$
where y_i' denotes the derivative value at x_i and b_i is a binary flag indicating whether or not the derivative interpolation condition is to be met.

2. Extend the LS linear regression approach to the following data fitting problem with data (x_i, y_i), $i = 0 : n$, subject to the equality constraints,
$$p(x_j) = y_j, \quad j \in S_n$$
where S_n is a small subset of $\{0, 1, \cdots, n\}$ and p is a polynomial interpolant.

3. Describe the difference in the update computation at each elimination step between the LU factorization with pivoting and the QR factorization of a matrix.

4. Find some common features and differences between the following two matrix operators
$$H_1 = I - uu^{\mathrm{T}}, \quad H_2 = I - 2uu^{\mathrm{T}}$$
where I is the identity matrix, u is a real vector with unit Euclidean length $u^{\mathrm{T}}u = 1$.

5. Describe the structures of the Laplacian matrices on a finite d dimensional grid, $d = 1, 2, 3$.

Part II

1. The Runge function is defined as follows.

$$\mathrm{runge}(x) = \frac{1}{1+x^2}, \quad x \in (-\infty, \infty).$$

   Compare the following interpolation methods on the interval $[-5, 5]$, as the number of interpolation nodes increases.

   (a) the Vandermonde approach

   (b) the Cubic spline approach

   (c) (**optional**) the B-spline approach

   Built-in or provided MATLAB functions or scripts :
   `pval, ppval, runge, rungeView, pPolyEval`

2. Provide review comments on Problem 4 of Exercise Set 3.5. in the textbook (parametric curves).