Study of the Poisson Equation

Model assumptions :

- the d-dimensional domain is the Kronecker product of d intervals with one for each dimension, $d = 1, 2, 3$;
- the Dirichlet boundary condition.

Discretization :

- the use of finite-difference discretization technique with the spacing parameter h;
- Establish and express the system of discretized equations in matrix form.

Numerical solution methods.

1. Design termination criteria for the use of iterative methods for solving the discrete Poisson system;
2. (a) Develop a routine for matrix-vector multiply that exploits the matrix structure;
   (b) Develop a routine for the block Jacobi iteration;
   (c) Develop a routine for the block Gauss-Seidel iteration;
   (d) Develop a routine for the Conjugate Gradient iteration;
3. Find examples in Chapter 12 for numerical testing and comparisons.
4. **Present comparisons** with different values of h, with the same set of criteria; specify the cost of computational operations.

Review of CG methods

Assume A is a symmetric and positive definite matrix.

* Inner product. Verify that $x^{\mathrm{T}}Ay$ is a well defined inner product.
* Orthogonalization.

  A procedure, and its verification, to generate A-conjugate vectors from $\{r, Ar, A^2r, \cdots\}$ with three terms recurrence and matrix vector products.
* Successive Least Square Solution in the introduced A-norm as every new A-conjugate vector becomes available.
* Error propagation analysis.