Today’s topics
Sponsored Search Markets
Acknowledgements
Notes from Nicole Immorlica & Jason Hartline

Advertising
• What’s the point of advertising in newspapers and TV?
  – How are prices for ad time/space determined?
• 2011 Super Bowl
  – 30 second ad cost $3 million
  – 111 million viewers
  – How are prices determine
• How are online ads different?
  – Metrics, targeting, market...
• How does Google make money?

Matching (per keyword)

Matching market
Click-through rates (# of clicks/hour)  Value per click

Slots  Advertisers

<table>
<thead>
<tr>
<th>10</th>
<th>4</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
</tbody>
</table>

Value per click

| 7 | 6 | 1 |
Value of advertiser j for slot i is:

\[ v_{ij} = \text{(value per click)} \times \text{(click-through rate)} \]

Market-clearing matching

<table>
<thead>
<tr>
<th>Slot</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7 x (10, 4, 0) = (70, 28, 0)</td>
</tr>
<tr>
<td>4</td>
<td>6 x (10, 4, 0) = (60, 24, 0)</td>
</tr>
<tr>
<td>0</td>
<td>1 x (10, 4, 0) = (10, 4, 0)</td>
</tr>
</tbody>
</table>

Market-clearing prices

<table>
<thead>
<tr>
<th>Value</th>
<th>Slot</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ 40 = 10 \times 4 \]
\[ 4 = 4 \times 1 \]
\[ 0 = 0 \times 0 \]

What if we don’t know values?

Run an auction.
Generalized Second Price Auction

Each advertiser $j$ announces a bid $b_j$

Slot $i$ is assigned to the $i^{th}$ highest bidder at a price per click equal to the $(i+1)^{st}$ highest bidder’s bid

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**How should you bid?**

*Experiment:*

- **Click-through rates**
  - A: [10] 7
  - B: [4] 6
  - C: [0] 1

- **Bid**
  - A: [1]
  - B: [2]
  - C: [3]

Write your name and your bid on your card. Point total = payoff / 10.

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**Truthful Bidding is Not Necessarily an Equilibrium!**

(and therefore also not a dominant strategy)

- **Click-through rates**
  - A: [10] 7
  - B: [4] 6
  - C: [0] 1

- **Bid**
  - A: [1]
  - B: [2]
  - C: [3]

If each bidder bids their true valuation, then A gets Slot 1 and her payoff is $7 \times 10 - 6 \times 10 = 10$
Truthful Bidding is Not Necessarily an Equilibrium! (and therefore also not a dominant strategy)

If A were to bid 5, then she gets Slot 2 and her payoff is $7*4-1*4=24$ (which is higher than 10!).

Click-through rates

<table>
<thead>
<tr>
<th>Bid</th>
<th>Value</th>
<th>Click-through rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

If A were to bid 5, then she gets Slot 2 and her payoff is $7*4-1*4=24$ (which is higher than 10!).

What are the “nice” equilibria?

**“Market-clearing equilibrium”**

<table>
<thead>
<tr>
<th>Bid</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 (= 10 * 4)</td>
<td>&gt; 4</td>
</tr>
<tr>
<td>4 (= 4 * 1)</td>
<td>4</td>
</tr>
<tr>
<td>0 (= 0 * 0)</td>
<td>1</td>
</tr>
</tbody>
</table>

Market-clearing prices/matching → players maximize payoff → no one wants to raise or lower bid
Is There a Way to Encourage Truthful Bidding?

Vickrey-Clarke-Groves Mechanism

Each individual is charged the harm they cause to the rest of the world

Second Price Sealed Bid Auctions Revisited

If bidders values in decreasing order were $v_1$, $v_2$, $v_3$, ..., $v_n$

Then bidder 1 would win

If bidder 1 were not present, the object would go to bidder 2, who values it at $v_2$

Bidders 2, 3, ..., n collectively experience a harm of $v_2$ because bidder 1 is there

What is the harm caused by bidder A’s existence?

If bidder A was not there, B would make 60 and C would make 4, which improves their combined valuation by 24. So A has to pay 40.
If items are assigned and prices computed according to the VCG procedure, then truthfully announcing valuations is a dominant strategy for each buyer, and the resulting assignment maximizes the total valuation of any perfect matching of slots and advertisers.

The economist as an engineer:

**Ad quality**: ad-dependent click-through rates

**Click fraud**: incentive-based machine learning

**Bidding language**: budgets, slot-dependent bids, enhanced targeting

**Competing platforms**: prevalence of Google’s “mistake”