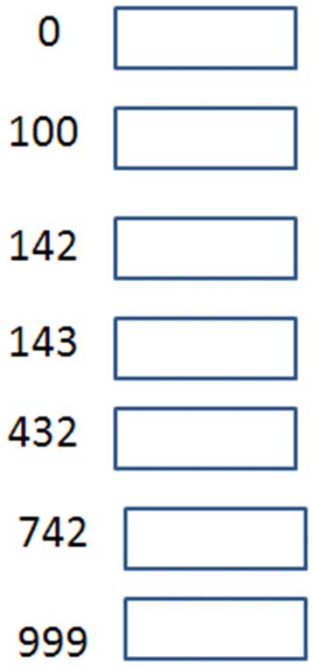
CompSci 100e
Program Design and Analysis II



February 8, 2011

Prof. Rodger

Announcements

- What is due?
  - Assignment Prestidigitation due today!
  - Apt due Feb 15
  - Markov – out today - Assignment due Feb 17
    - Lab this week also on Markov
  - Exam Feb 22
- Always turn in APTs if you have any green! You can get partial credit.
- Each APT is 10 points. You can do extra APTs to make up the missing points.

Classwork 5

- Redo ClassScores with a Map

- BTW, We are reworking some of the APTs and now to run APT you cannot have in your class

  import java.io.*

  – That means that your Classwork 2 with classscores no longer works on the APT. You have to remove the import and the methods that use it to get it to run

Analysis – Data Structures and Algorithms

- How do we compare two programs?
- Which one will run faster? Can we tell before coding?

- Two ways:
  - Run them and see which is faster – add timings
  - Use mathematics to analyze the algorithm

Quantitative Measurements of Code

- Typically measure running time (memory?)
  - Other things to measure?
- Typically change size of input/problem to validate runtime hypotheses
  - Not the data itself, but the number of data items
  - Size of string vs. number of strings in array?
- Doubling hypothesis: What effect does doubling input size have on running time?
  - Linear: time doubles, quadratic: factor of four, …

Different measures of complexity

- Worst case
  - Gives a good upper-bound on behavior
  - Never get worse than this
  - Drawbacks?
- Average case
  - What does average mean?
  - Averaged over all inputs? Assuming uniformly distributed random data?
  - Drawbacks?
- Best case
  - Linear search, useful?

Notations for measuring complexity

- O-notation or big-Oh: what does n look like as n approaches infinity?
  - O(n) linear,
  - $O(n^2)$ quadratic
  - $O(n^3)$ cubic
  - O(log n) logarithmic
  - $O(2^n)$ exponential
- Sedgewick/Wayne uses tilde notation ~ n^2 means leading term is n squared

Example

```
  for (int k=0; k<n; k++)  {
        for (int j=0; j<n; j++)  {
              count ++;
        }
}
```

What is O() worst case?

Example

```
  for (int k=1; k<n; k = k * 2)  {
        for (int j=0; j<m; j++)  {
              count ++;
        }
}
```

What is O() worst case?

Example

```
  for (int k=0; k<n; k++)  {
        for (int j=k; j<n; j++)  {
              count ++;
        }
}
```

What is O() worst case?

Big-Oh, O-notation: concepts & caveats

- Count how many times “simple” statements execute
  - In the body of a loop, what matters? (e.g., another loop?)
  - Assume simple statements take a second, cost a penny,…
- In real life: cache behavior, memory behavior, swapping behavior, library gotchas, things we don’t understand,…

Simplify

$O(4n^3 + 100\,n + 35) =$

$O(3n^2 + 4n + 6) =$

Worst case

- Given an array of integers in sorted order
- What is the worst case big-Oh time for:
  - Insert(x) - insert x in the array (must maintain the sorted property.
  - Delete(x) – remove x from the array
  - Contains(x) - is x in the array, T or F
  - Min(x) – return the minimum value in the array (don't delete it)

Multiplying and adding big-Oh

- Suppose we do a linear search then do another one
  - What is the complexity?
  - If we do 100 linear searches?
  - If we do n searches on a vector of size n?

- Binary search followed by linear search?
  - What are big-Oh complexities? Sum?
  - What about 50 binary searches? What about n searches?

- What is the number of elements in the list (1,2,2,3,3,3); (1,2,2,3,3,3,4,4,4,4)?
  - What about (1,2,2, …, n,n,…,n)?

Helpful formulae

- We always mean base 2 unless otherwise stated
  - What is log(1024)?
  - $\log(xy)$ $\log(x^y)$ $\log(2^n)$ $2^{(\log n)}$

- Sums (also, use sigma notation when possible)
  - $1 + 2 + 4 + 8 + \ldots + 2^k =$
  - $1 + 2 + 3 + \ldots + n =$
  - $a + ar + ar^2 + \ldots + ar^{n-1} = a(r^n - 1)/(r-1) = \sum_{i=0}^{n-1} ar^i$

Helpful formulae

- We always mean base 2 unless otherwise stated
  - What is log(1024)?
  - $\log(xy)$ $\log(x^y)$ $\log(2^n)$ $2^{(\log n)}$

    - **$\log(x) + \log(y)$**
    - **$y \log(x)$**
    - **$n \log(2) = n$**
    - **$2^{(\log n)} = n$**
- Sums (also, use sigma notation when possible)
  - $1 + 2 + 4 + 8 + \ldots + 2^k =$ $2^{k+1} - 1 =$ $\sum_{i=0}^{k} 2^i$
  - $1 + 2 + 3 + \ldots + n =$ $n(n+1)/2 =$ $\sum_{i=1}^{n} i$
  - $a + ar + ar^2 + \ldots + ar^{n-1} = a(r^n - 1)/(r-1) =$ $\sum_{i=0}^{n-1} ar^i$

Hashing

- Storage – hash table or hash buckets (implementation could be array, map, tree, or other structure)
- Each data item has a key
- Hash function that maps key to address in the hash table H(key) = address
- Best if the “key” is near the address
- “collision” if two items map to the same address

Example Hashing

- Duke's ACM Chapter wants to be able to quickly find out info about its members. Also add, delete and update members. Doesn't need members sorted.

  267-89-5432 John Smith

  703-25-6142 Jack Adams

  319-86-2100 Betty Harris

  476-82-5142 Rose Black

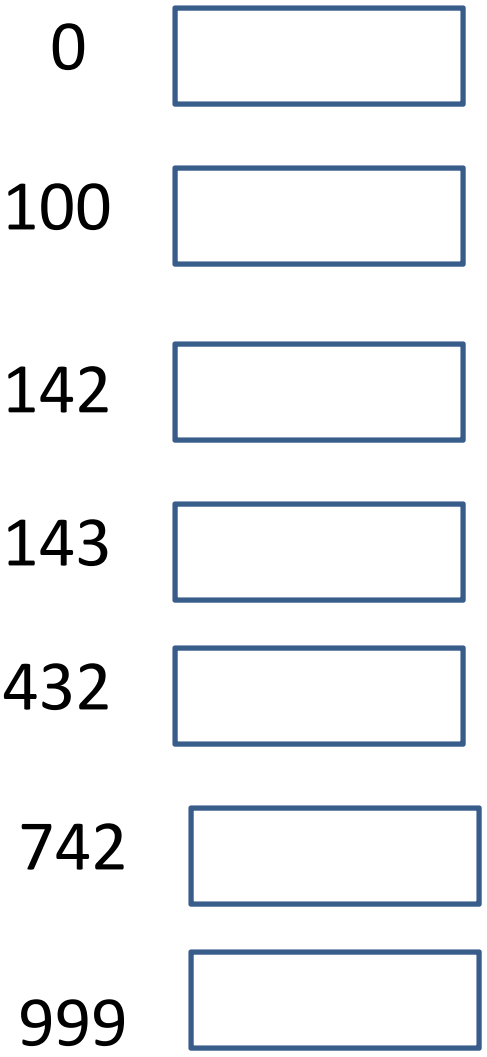
- Possible Hash Function: H(ssn) = last 3 digits
- Hash Table size is 1000, range from 0 to 999
- Will there be a collision with data above?

Bucket Hashing

Buckets 0-999 (not all buckets shown)

- Buckets hold a lot of values
- Apply hash function to data. What happens?

H(267-89-5432) = 432
John Smith
H(703-25-6142) = 142
Jack Adams
H(319-86-2100)= 100
Betty Harris
H(476-82-5142) = 142
Rose Black



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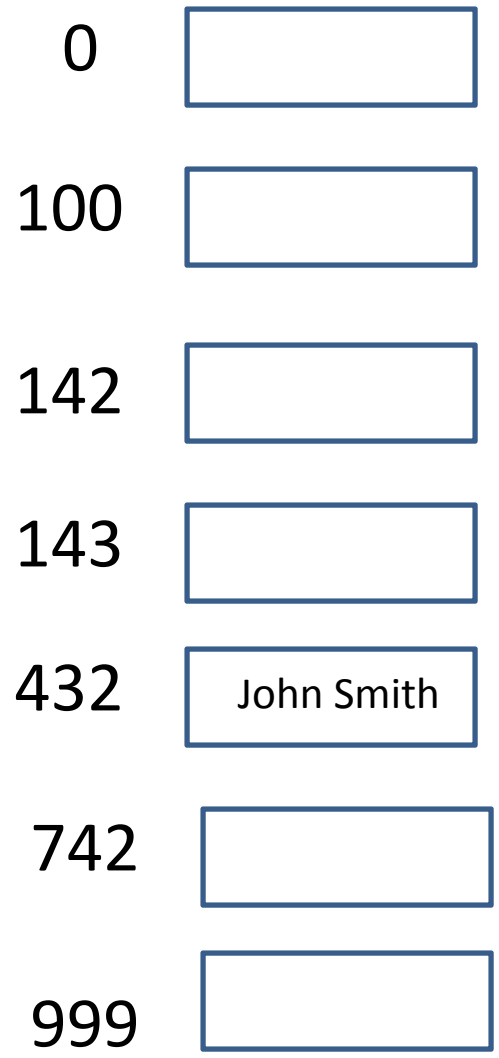

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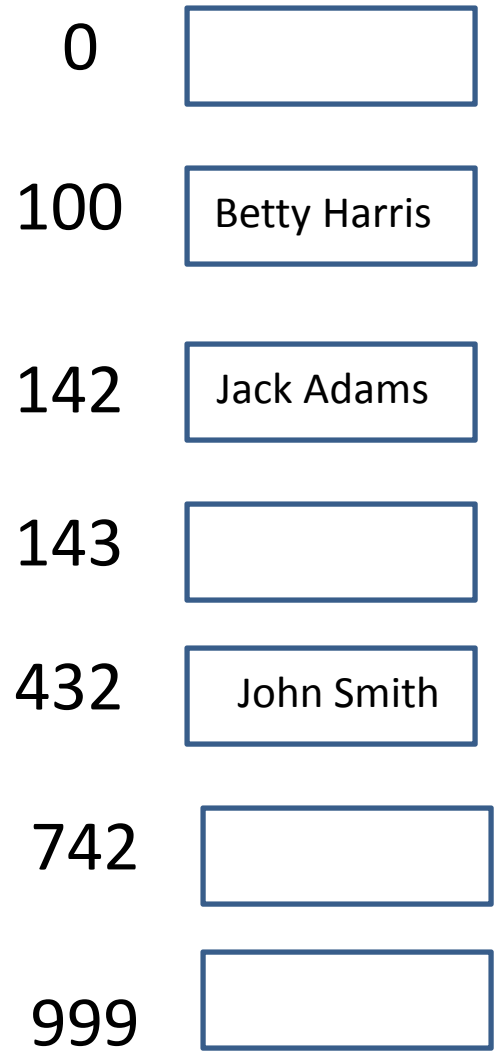
Where does Rose Black go?



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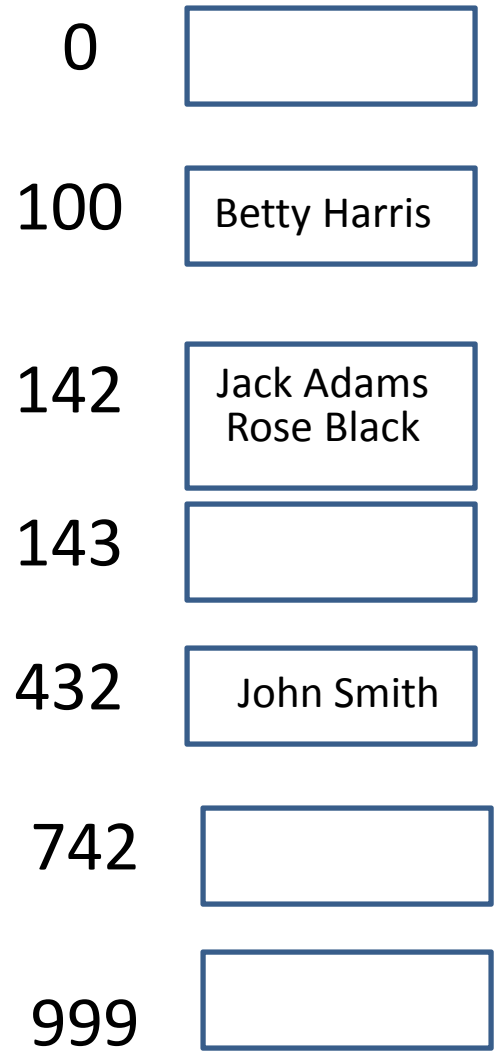
- If buckets are one array, where does Rose Black go?



If instead of Buckets, one String array 0-999, Where would Rose Black go?

- Collision, must be resolved, one way – next possible bucket (slot)
- Other Collision resolution methods

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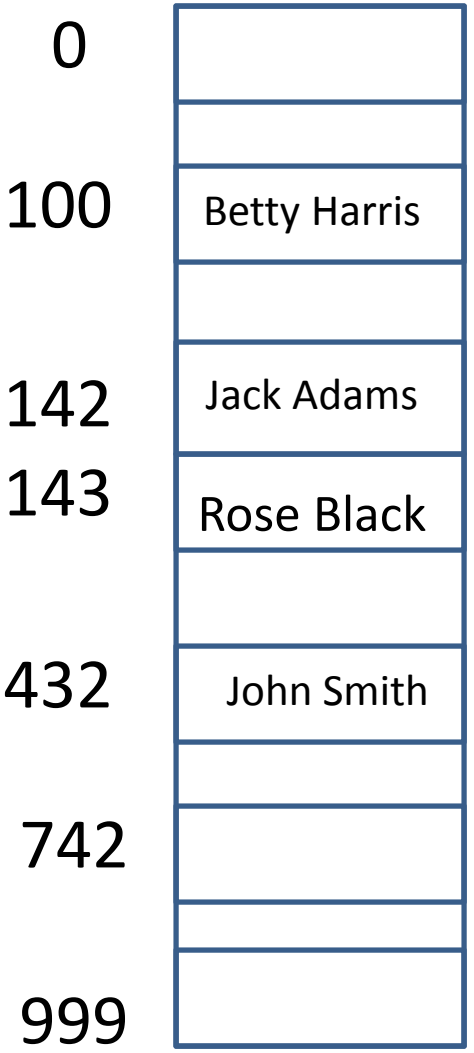

Hash Functions

- Want hash function with the fewest collisions, data distributed
- Random number Generator with seed
  - H(key) is random(key)*N
- Shift folding with 100 buckets
  - H(123-45-6789) is (123 + 45+ 6789) mod 100
- Use ascii code
  - H(BANKS) = 66 + 65 + 78 + 75 + 83 = 367
  - Where ascii(B) = 66, etc.
- Java – hashCode()