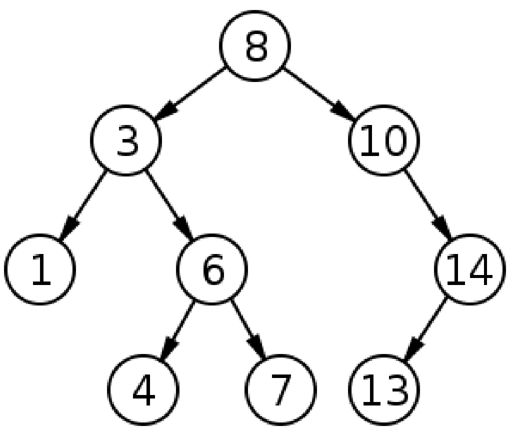
CompSci 100e
Program Design and Analysis II



March 29, 2011

Prof. Rodger

Announcements

- One APT next week – BSTCount
  - Will do in class
- Written Assignment lists/trees due March 31
- New assignment Boggle due April 7
  - Will do part of it in lab (last time, and next lab)
- Today
  - More on trees and analysis with trees
  - Recurrence relations

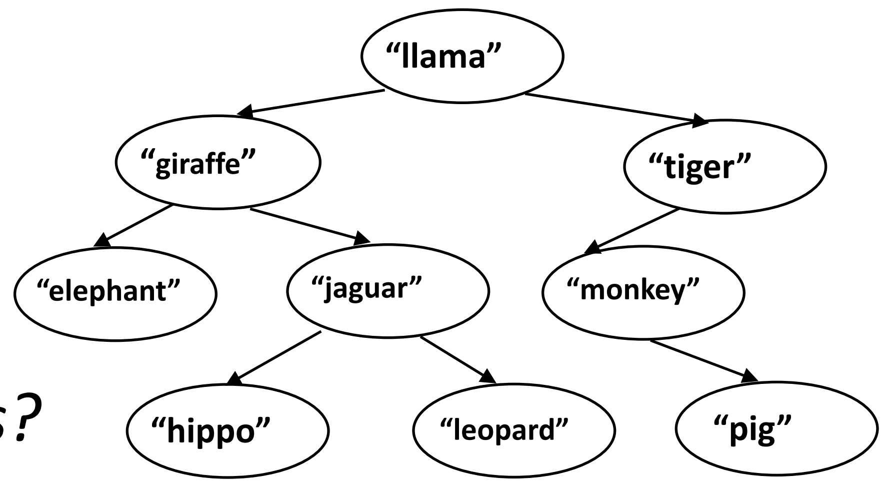
More on Trees

- Focus on binary trees
  - Includes binary search trees
  - Process tree: root (subtree) (subtree)
  - Analyze recursive tree functions
    - Recurrence relation

Review: Printing a search tree in order

- When is *root* printed?
  - After left *subtree*, before right *subtree*.

```
void visit(TreeNode t){
    if (t != null) {
          visit(t.left);
          System.out.println(t.info);
          visit(t.right);
    }
}
```



- *Inorder traversal*
- *How long for n nodes?*
  - *O()?*

Tree functions

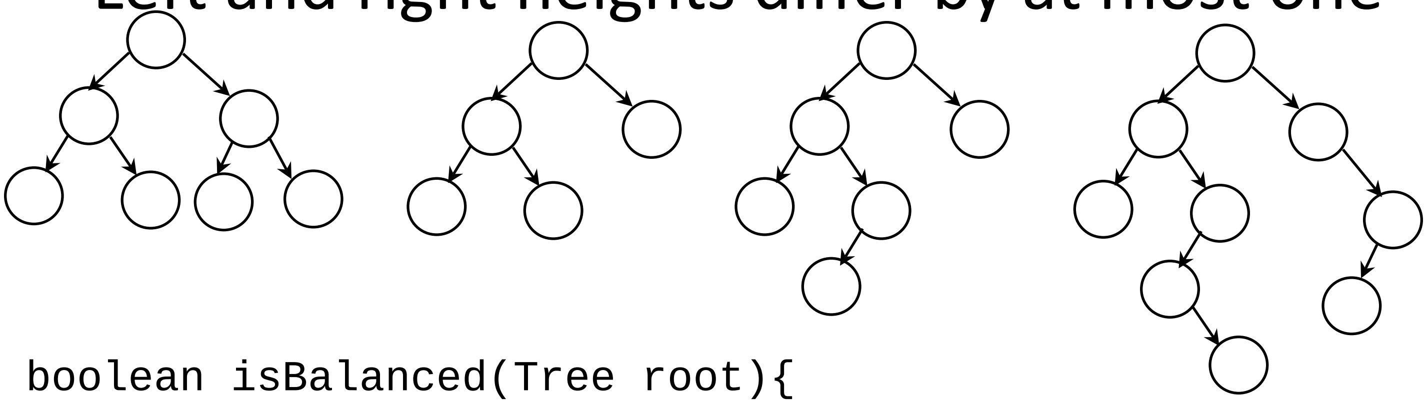
- Compute height of a tree, what is complexity?

```
int height(Tree root) {
    if (root == null) return 0;
    else {
       return 1 + Math.max(height(root.left),
                           height(root.right) );
    }
}
```

- Modify function to compute number of nodes in a tree, does complexity change?
  - What about computing number of leaf nodes?

Balanced Trees and Complexity

- A tree is height-balanced if
  – Left and right subtrees are height-balanced
  – Left and right heights differ by at most one



```
boolean isBalanced(Tree root){
   if (root == null) return true;
   return
      isBalanced(root.left) && isBalanced(root.right) &&
      Math.abs(height(root.left) – height(root.right)) <= 1;
     }
}
```

What is complexity?

- Consider worst case? What does the tree look like?
- Consider average case? Assume trees are “balanced” in analyzing complexity
  - Roughly half the nodes in each subtree
  - Leads to easier analysis

- How to develop recurrence relation?
  - What is T(n)?
  - What other work is done?

- How to solve recurrence relation – formula for recursion
- Plug, expand, plug, expand, find pattern
  - A real proof requires induction to verify correctness

Solving Recurrence Relation

- Recurrence relation is a formula that models how much time the method takes.
- T(n) – the time it takes to solve a problem of size n
- Basis – smallest case you know how to solve, such as n=0 or n=1
- If two recursive calls formula might be:
  - T(n) = T(smaller problem) + T(smaller problem) + work to put answer together…
- On the right side, replace T(smaller) by plugging it in to the formula

Solving Recurrence Relation (cont)

- Continue replacing the T(smaller) values until you see a pattern – use k for the pattern
- Then solve for k with respect to N to get a basis case that has a constant value – this removes the T term from the right hand side of the equation and you are left with T(N) = to terms of N and can easily compute big-Oh

What is average big-Oh for height?

- Write a recurrence relation
- T(0) =
- T(1) =
- T(n) =

What is worst case big-Oh for height?

- Write a recurrence relation
- $T(0) =$
- $T(1) =$
- $T(n) =$

What is average case big-Oh for is-balanced?

- Write a recurrence relation
- T(1) =
- T(n) =

Recognizing Recurrences

- Solve once, re-use in new contexts
  - T must be explicitly identified
  - n must be some measure of size of input/parameter
    - T(n) is for quicksort to run on an n-element array

`T(n) = T(n/2) + O(1)`	binary search	`O( )`
`T(n) = T(n-1) + O(1)`	sequential search	`O( )`
`T(n) = 2T(n/2) + O(1)`	tree traversal	`O( )`
`T(n) = 2T(n/2) + O(n)`	quicksort	`O( )`
`T(n) = T(n-1) + O(n)`	selection sort	`O( )`

- Remember the algorithm, re-derive complexity

Recognizing Recurrences

- Solve once, re-use in new contexts
  - T must be explicitly identified
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    - T(n) is for quicksort to run on an n-element array

`T(n) = T(n/2)   + O(1)`	binary search	`O(`	**log n**)
`T(n) = T(n-1)   + O(1)`	sequential search	`O(`	**n**)
`T(n) = 2T(n/2) + O(1)`	tree traversal	`O(`	**n**)
`T(n) = 2T(n/2) + O(n)`	quicksort	`O(`	**n log n**)
`T(n) = T(n-1)   + O(n)`	selection sort	`O(`	**n^2**)

- Remember the algorithm, re-derive complexity

BSTCount APT

- Given values for a binary search tree, how many unique trees are there?
  - 1 value = one tree
  - 2 values = two trees
  - 3 values = 5 trees
  - N values = ? trees
- Will memoize help?

Recurrences

- If T(n) = T(n-1) + O(1)… where do we see this?

```
T(n) = T(n-1) + O(1)
```

true for all X so, `T(n-1) = T(n-2)+ O(1)`

```
T(n) = [T(n-2) + 1] + 1 = T(n-2) + 2
     = [T(n-3) + 1] + 2 = T(n-3) + 3
```

- True for 1, 2, so eureka! We see a pattern

`T(n) = T(n-k) + k,` true for all `k,` let n=k

```
T(n) = T(n-n) + n = T(0) + n = n
```

- We could solve, we could prove, or remember!