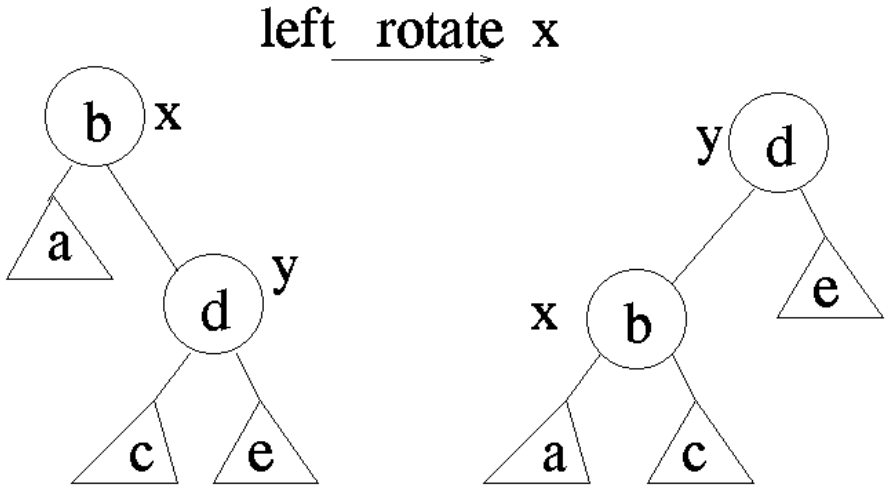
# CompSci 100e
# Program Design and Analysis II




April 21, 2011

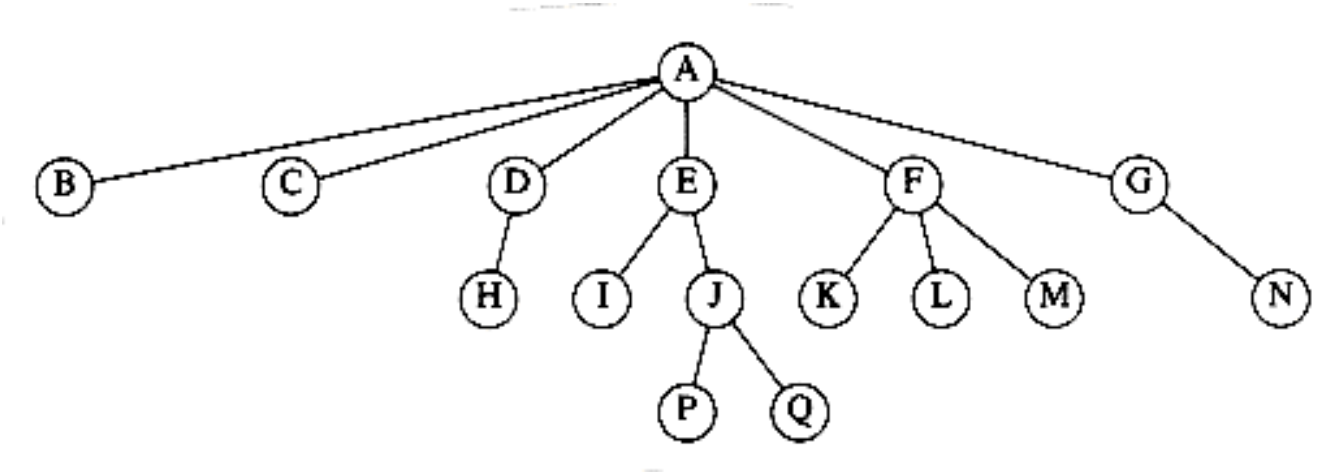
Prof. Rodger

# Announcements

- Lab tomorrow
  - Focus on BoggleScore APT
- What is due:
  - Huffman (extended til Monday night, Apr 25)
  - APTS due Tue, April 26
  - Extra credit assignments due Wed, April 27
- Graphs and Red-Black trees today
  - Classwork 16
  - Internet APT – work through parts in class

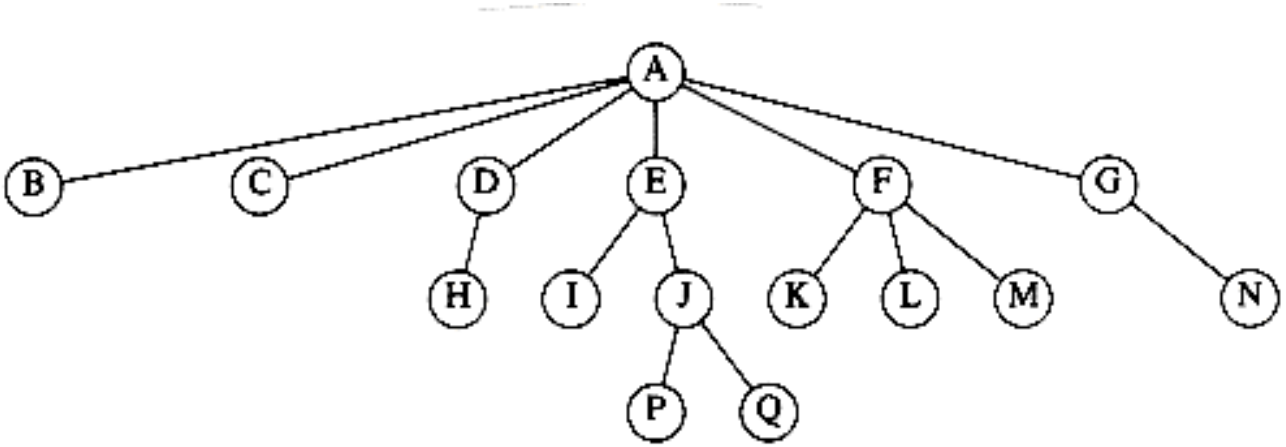
# More on trees

- General tree

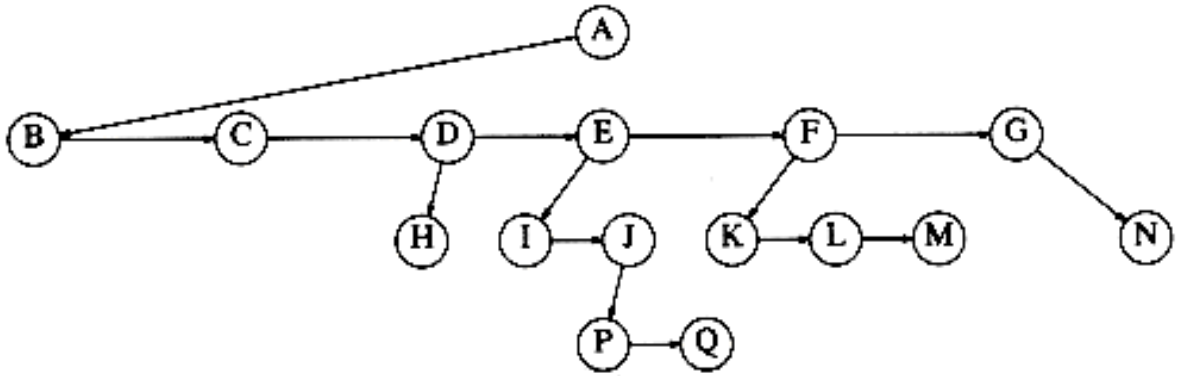


# More on trees

- General tree




- Implementation



# Balanced Trees

- Splay trees
- AVL trees
- Red-black trees
- B-trees

# Balanced trees, we'll look at red-black

- Both kinds have worst-case O(log n) time for tree operations
- AVL (Adel'son-Velskii and Landis), 1962
  - Nodes are "height-balanced", subtree heights differ by 1
  - Rebalancing requires per-node bookkeeping of height
  - http://people.ksp.sk/~kuko/bak/

- Red-black tree uses same rotations, but can rebalance in one pass, contrast to AVL tree
  - In AVL case, insert, calculate balance factors, rebalance
  - In Red-black tree can rebalance on the way down, code is more complex, but doable
  - Standard `java.util.TreeMap/TreeSet` use red-black

# Red-Black Tree

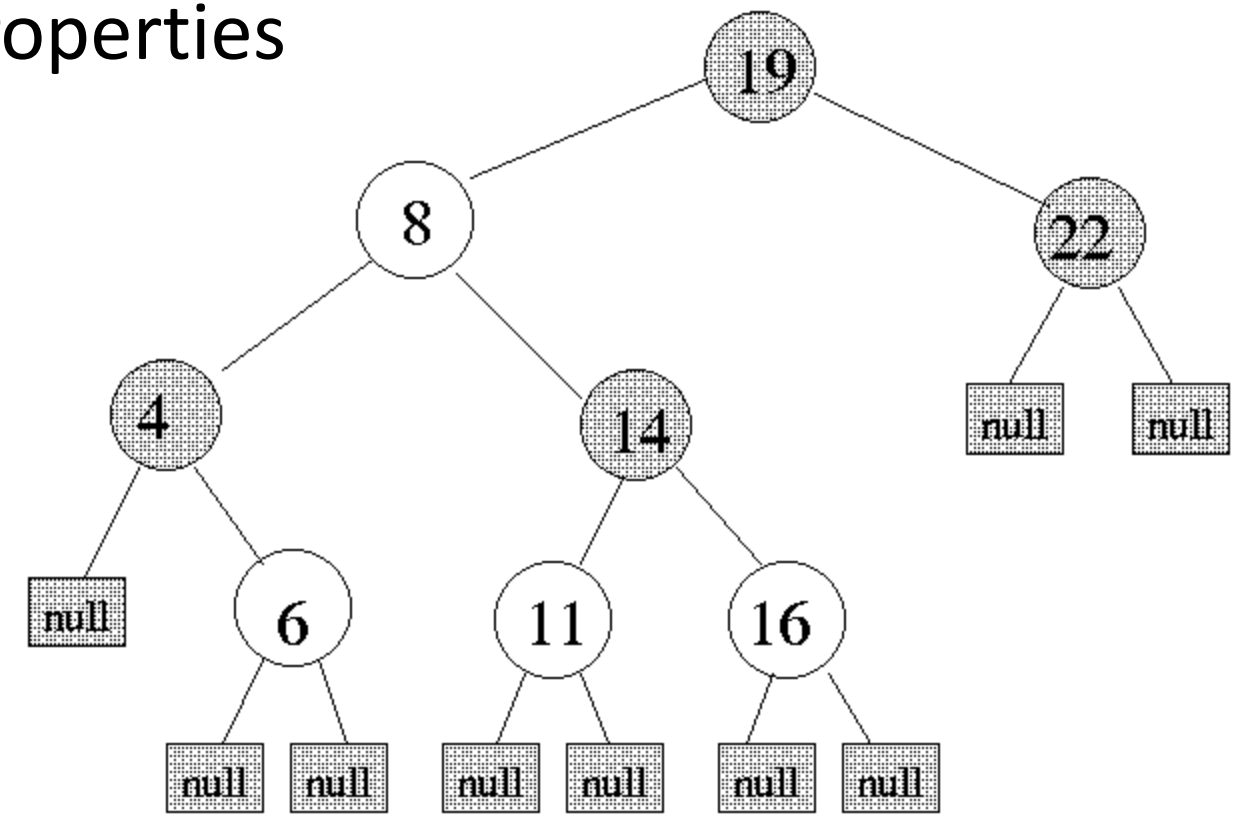
- Invented by Bayr – (though called them something else)
- Robert Tarjan (Turing Award Winner) – noticed the rotations were O(1)
- Type of balanced tree – uses color scheme, recoloring and rotations to balance

# Red-Black Tree

- Is a Binary Search Tree
- Properties:
  - Every node is red or black
  - The root is black
  - If a node is red, then its children are black
  - Every leaf is a null node and black (external node)
  - Every simple path from a node to a descendant leaf contains the same number of black nodes.

# Example red-black tree

- In the figure, black nodes are shaded and red nodes are non-shaded
- Check properties



# Example

- The five properties ensure that no path is more than twice as long as any other path
- Def. The *height* (h) of a node is the length of the longest path from the node (downward) to a leaf (including external nodes).
- Def. The *black height* (bh) of a node x is the number of black nodes on any path from x (not including x) to a leaf
- Examples: h(19)? bh(19)? h(8)? bh(8)?

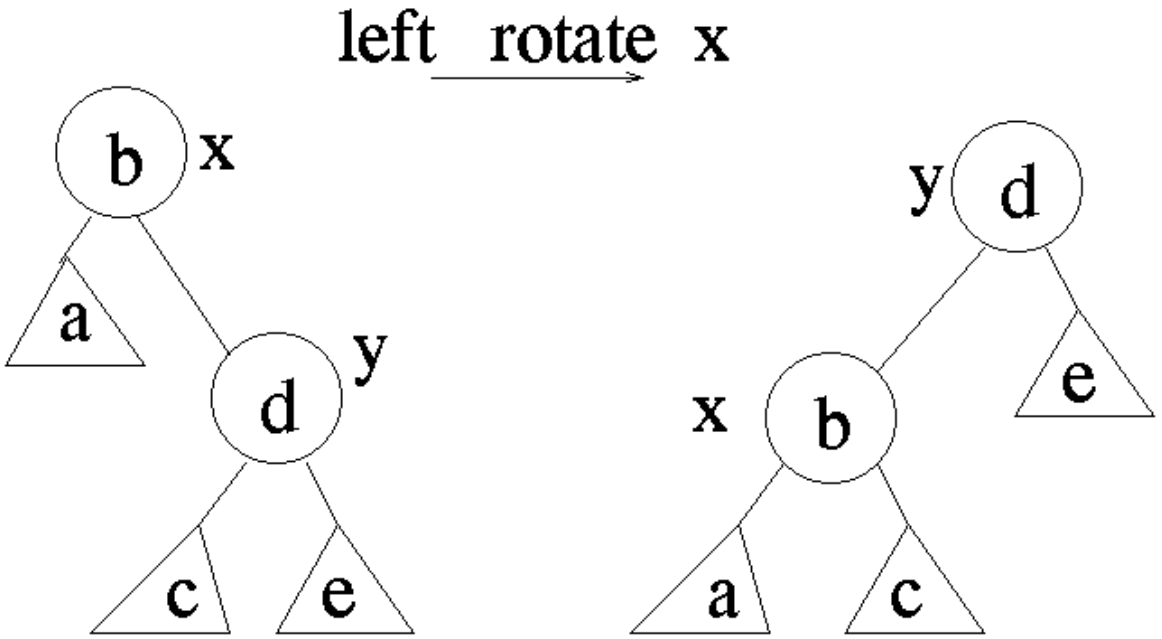
# Height of Red-Black Tree

- Lemma: A red-black tree with n internal nodes has height at most 2 log (n+1)

- Operations:
  - Time for search for x :
  - Time for min:
  - Time for list inorder:

# Rotations

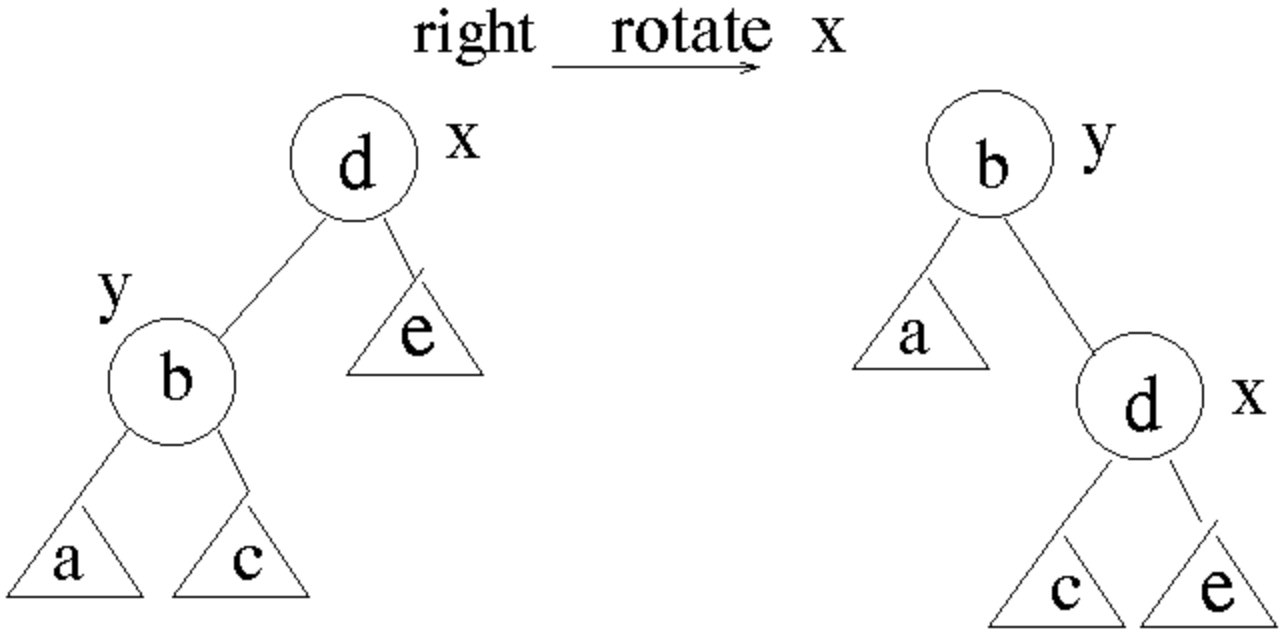
- We want to perform insertions and deletions in O(log n) time. Adding or deleting a node may disrupt one of its properties, so in addition to some recolorings, we may also have to restructure the tree by performing a rotation (change some pointers).



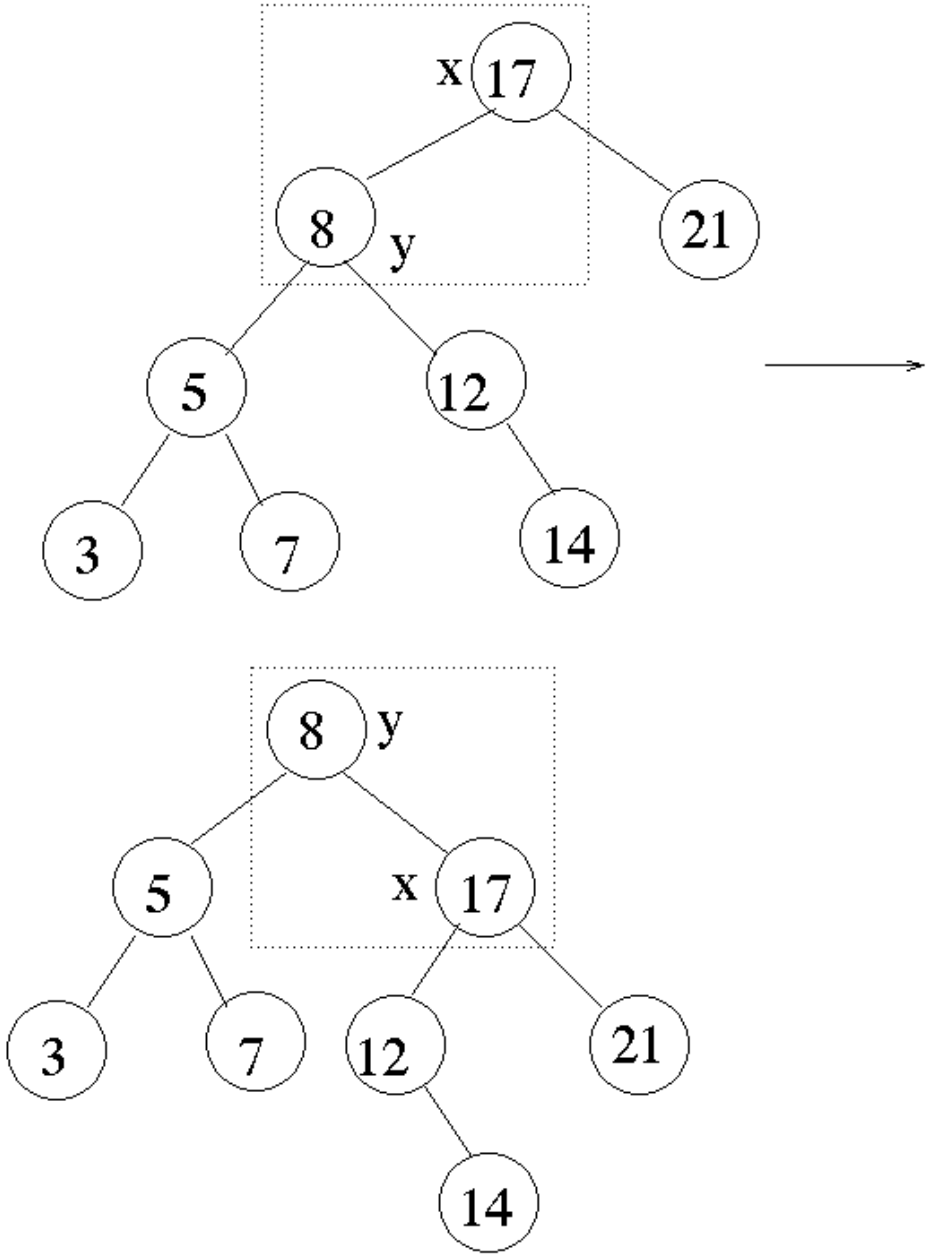

- Note the inorder traversal in both is: abcde

# Right Rotate

- Note the rotations change the pointer structure while preserving the inorder property.



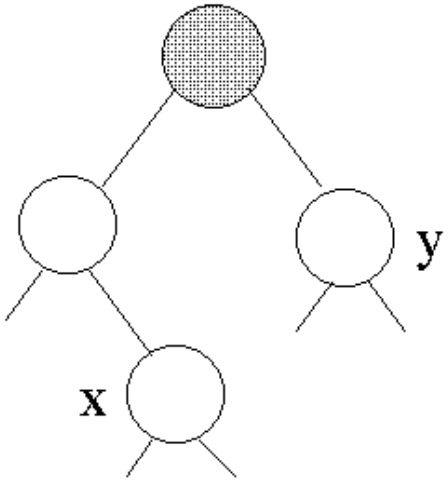
# Example of rotation



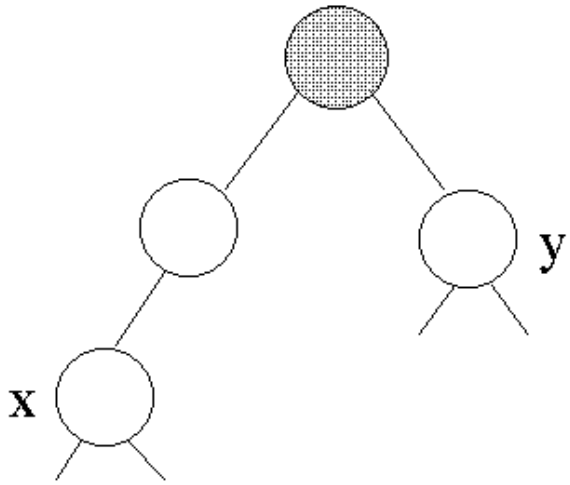
# Insertion

- Insert node as RED using a binary search tree insert
  - Means insert as a Red leaf with two black NULL nodes
- Then fix-up so that properties still hold
  - Recoloring and/or 1-2 rotations
- Several cases to consider

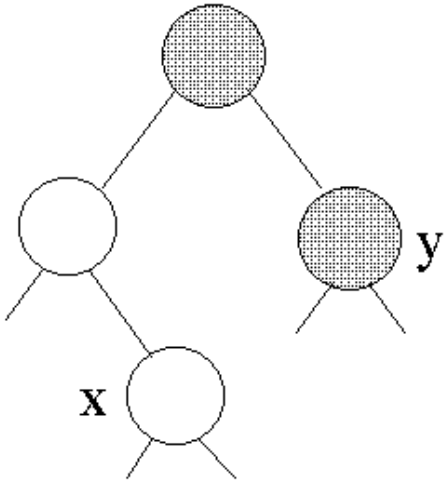
# Cases for Insert



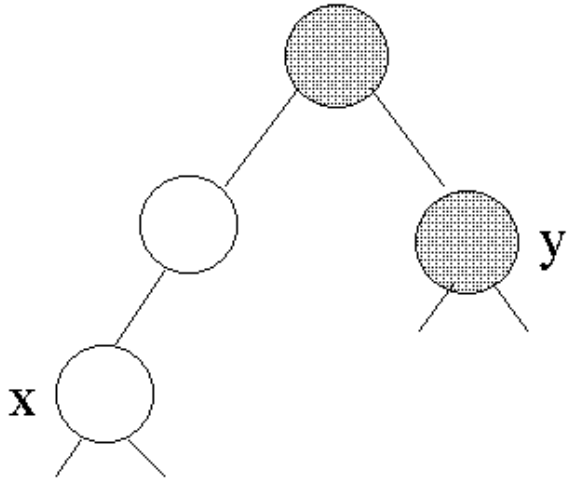

case 1




case 1



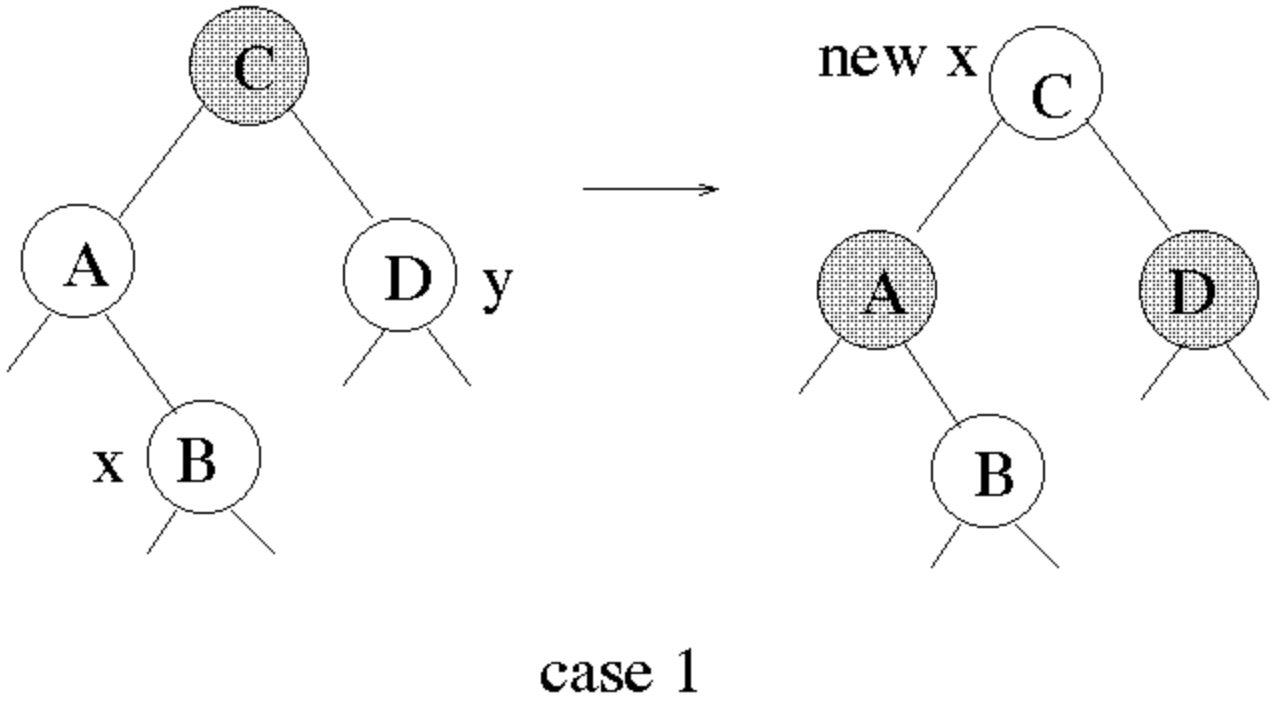

case 2




case 3

# Insertion – Case 1 – How to Fix

- Case 1 – sibling of parent of x (called y) is red

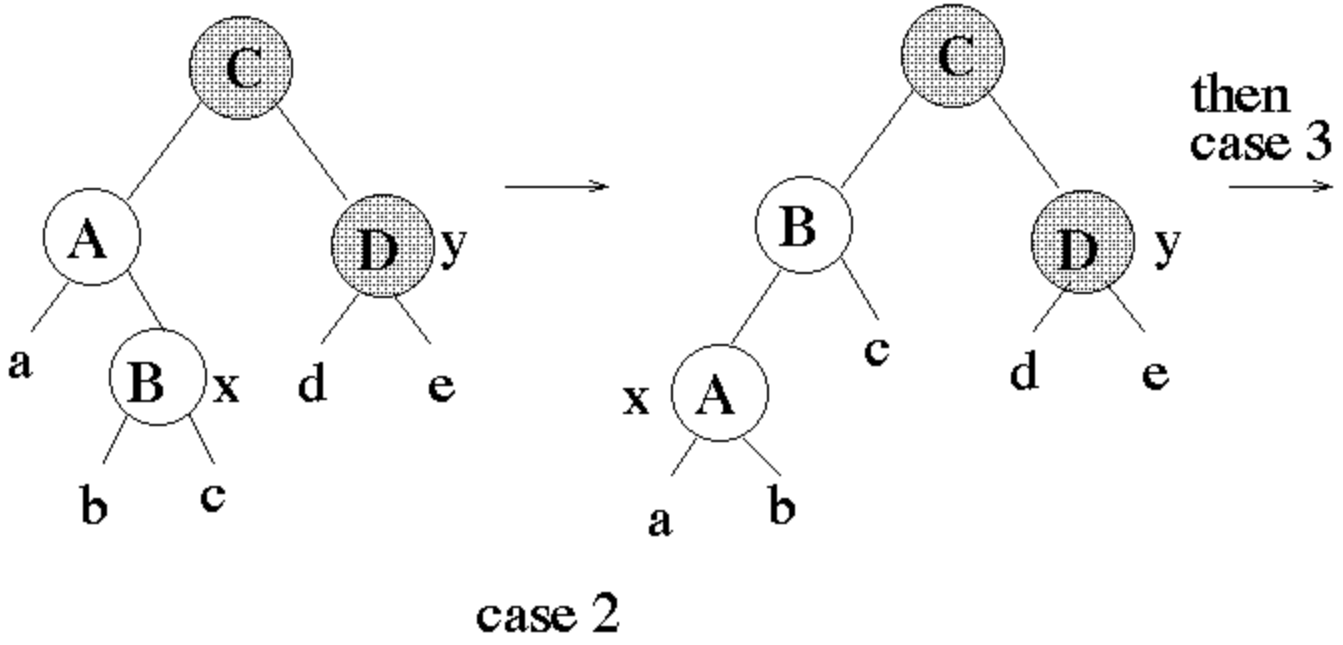



case 1

- To fix: recolor three nodes, then fix up new "x"

# Insertion – Case 2 How to Fix

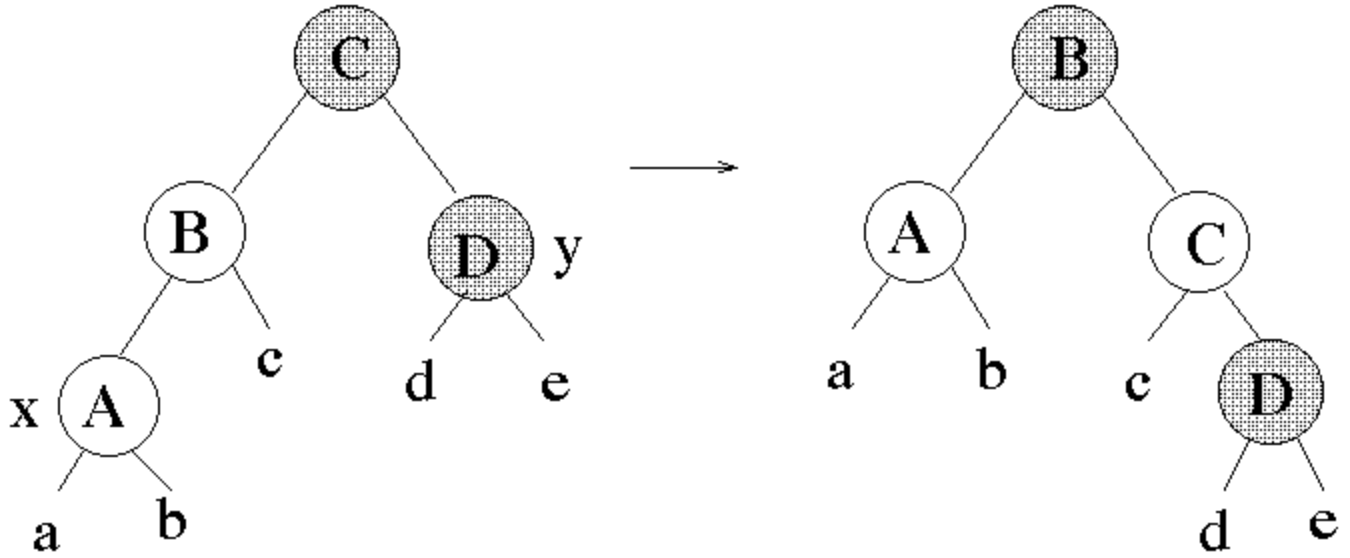
- Sibling of parent of x (call y) is black, x right child




- To fix: set x to parent of x and left rotate x, then it becomes a case 3

# Insertion – Case 3 – How to Fix

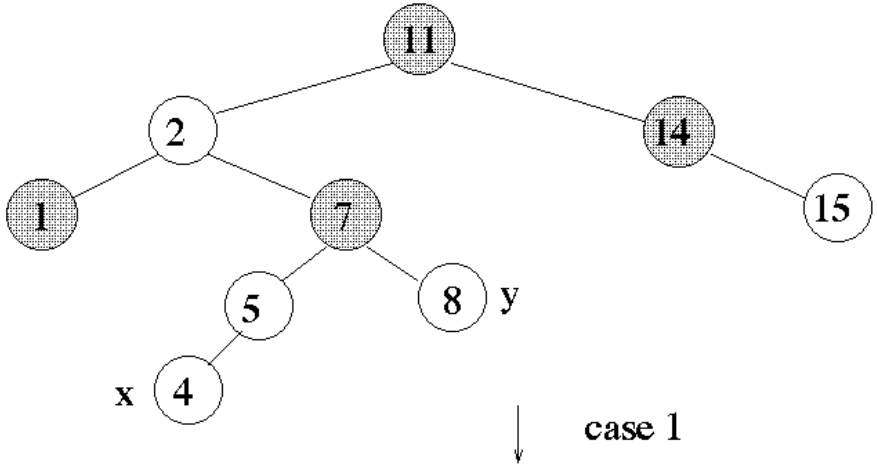
- Case 3 – sibling of parent of x (call y) is black, x left child



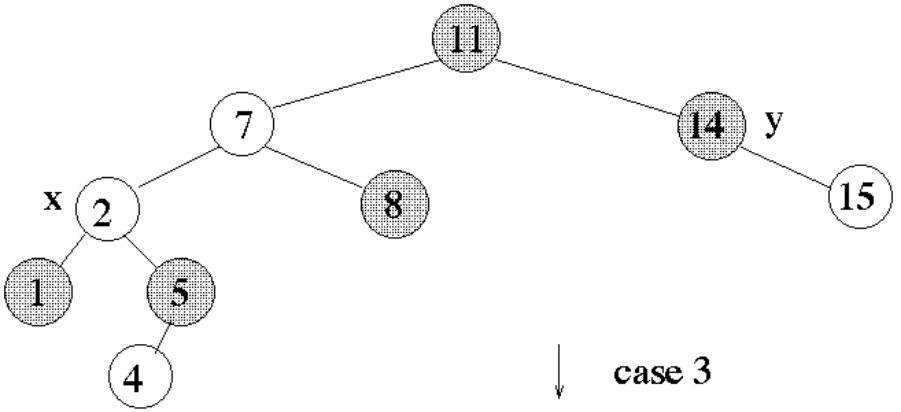

case 3

- To fix: two recolorings and one right rotate of grandparent of x

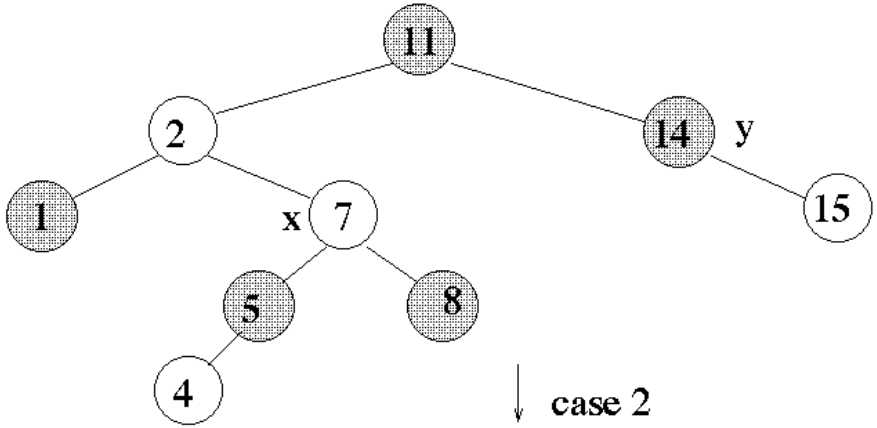
# Example of Insert 4 w/ double rotation



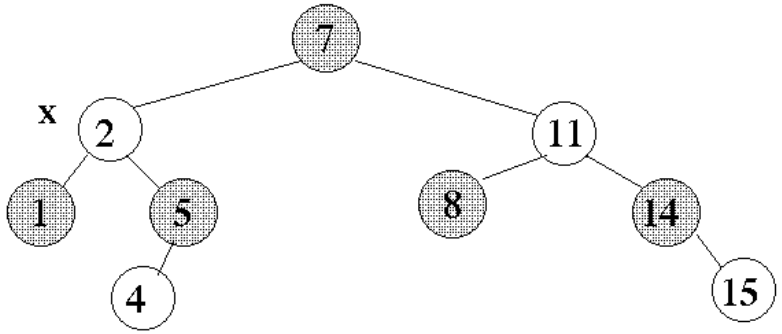

case 1




case 3




case 2



# Analysis – Red Black Tree

- Insert
- Deletion

# Analysis – Red Black Tree

- Insert O(log n)
- Deletion O(log n)