1. (10 points) Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

2. (15 points) Show that $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology.

3. (15 points) Show that if A and B are finite sets, then $|A \cap B| \leq |A \cup B|$. Determine when this relationship is an equality.

4. (10 points) Let A and B be subsets of the finite universal set U. Show that

$$|\bar{A} \cap \bar{B}| = |U| - |A| - |B| + |A \cap B|.$$

5. Suppose that the sequence x_1, x_2, $\ldots$, x_n, $\ldots$ is recursively defined by $x_1 = 0$ and $x_{n+1} = \sqrt{x_n + 6}$.
   (a) (10 points) Use mathematical induction to show that $x_1 < x_2 < \cdots < x_n < \cdots$, that is, the sequence $\{x_n\}$ is monotonically increasing.
   (b) (10 points) Use mathematical induction to show that $x_n < 3$ for $n = 1, 2, \ldots$.

6. You have a scale and two piles of weights: 3oz weights and 5oz weights.
   (a) (5 points) Find integers a and b such that $3a + 5b = 1$.
   (b) (10 points) Show that, if you are allowed to place weights on both sides of the scale then you can measure any integer number of ounces.
   (c) (15 points) If you can only put weights on one side then show that you can collect n ounces on side for all $n \geq 8$. [*Hint: Study Example 4 of Section 4.2, P. 287*]

7. (10 points) **(Bonus)** Show that if S is a set, then $|S| < |P(S)|$. [*Hint: Exercise #48, P. 163*]