Lecture 13, Feb 24, 2011, Binomial Coefficients

Theorem 1.

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

Theorem 2.

$$\binom{n+m}{r} = \sum_{k=0}^{r} \binom{m}{r-k}\binom{n}{k}$$

Theorem 3. *Let x and y be variables, and let n be a nonnegative integer. Then*

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

Example 1. *Permutations with repetition.*

Example 2. *How many ways are there to select five bills from a cash box containing \$1 bills, \$2 bills, \$5 bills, \$10 bills, \$20 bills, and \$100 bills? Assume that the order in which the bills are selected does not matter, that the bills of each denomination are indistinguishable, and that there are at least five bills of each type.*

Theorem 4. *There are $C(n+r-1, r)$ r-combinations from a set with n elements when repetition of elements is allowed.*

Example 3. *How many solutions does the equation*

$$x_1 + x_2 + x_3 = 11$$

have, where x_1, x_2, and x_3 are nonnegative integers? If we further require x_1 is an integer greater than or equal to 2, then what is the solution?

Recommended reading: section 5.1, 5.3, 5.4, and 5.5.

For section 5.1, tree diagrams were not discussed.

For section 5.5, we have not covered permutation (combination) with indistinguishable objects yet.

Practice problems: (suggested but not collected) §