LECTURE 21, GRAPH

Read §9.1.

simple graph: no two edges connected the same pair of vertices. Otherwise, the graph is called a multigraph. *Loops* are edges that connect a vertex to itself. *directed* and *undirected* graphs.

Two vertices u and v in an undirected graph G are called adjacent if u and v are endpoints of an edge in G. If e is associated with $\{u, v\}$, the edge e is called incident with u and v.

The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex that contributes twice to the degree of that vertex. Notation $\deg(v)$. If $\deg(v) = 0$, v is called isolated. If $\deg(v) = 1$, v is pendant.

Subgraph: $H = (W, F)$, where $W \subset V$, and $F \subset E$.

Special graphs:

(1) Complete graph K_n: exactly one edge between each pair of distinct vertices.
(2) Cycles: $\{v_1, v_2, \ldots, v_n\}$ contains (v_1, v_2), (v_2, v_3), $\ldots$, (v_n, v_1).
(3) Wheels: add an additional vertex and connect it with each of the n vertices in C_n.
(4) Bipartite graphs: $V = (V_1, V_2)$, V_1 and V_2 are disjoint such that each edge in G connects a vertex in V_1 and a vertex in V_2.

Theorem 1. *A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph such that no two adjacent vertices are assigned the same color.*

Theorem 2. *Let $G = (V, E)$ be an undirected graph with e edges, then*

$$2e = \sum_{v \in V} \deg(v)$$

Theorem 3. *An undirected graph has an even number of vertices of odd degree.*

Theorem 4. *If the graph is directed,*

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

Representing graphs using adjacency list, adjacency matrices, and incidence matrices.