Due Date: Thursday, March 29, 2012

1 Greedy Algorithm (20 points)

Consider the following single machine preemptive scheduling problem. We are given a set of n jobs $1, 2.....n$, with each job having a processing length $p_1, p_2.....p_n$ and release date $r_1, r_2....r_n$. The job i can be scheduled only after its release date r_i. Completion time of job i. denoted by C_i, is the earliest time t at which job is completely processed by the machine. Design an $O(n \log n)$-time algorithm to find a schedule that minimizes the total completion time, i.e., minimizes $\sum_i C_i$.

2 Dynamic Programming, DPV 6.1 (20 points)

A contiguous subsequence of a list S is a subsequence made up of consecutive elements of S. For instance, if S is 5, 15, -30, 10, -5, 40 10; then 15, 30, 10 is a contiguous subsequence but 5, 15, 40 is not. Give a linear-time algorithm for the following task:
Input: A list of numbers, $a_1, a_2; \dots a_n$.
Output: The contiguous subsequence of maximum sum (a subsequence of length zero has sum zero).
Hint: *For each $j \in \{1, 2 \dots n\}$ consider contiguous subsequences ending exactly at position j.*

3 Dynamic Programming, DPV 6.3 (20 points)

Yuckdonald's is considering opening a series of restaurants along Quaint Valley Highway (QVH). The n possible locations are along a straight line, and the distances of these locations from the start of QVH are, in miles and in increasing order, $m_1, m_2, \dots m_n$. The constraints are as follows:

- At each location, Yuckdonald's may open at most one restaurant. The expected profit from opening a restaurant at location i is p_i, where $p_i > 0$ and $i = 1, 2, \dots n$.
- Any two restaurants should be at least k miles apart, where k is a positive integer.

Give an efficient algorithm to compute the maximum expected total profit subject to the given constraints.

4 More Dynamic Programming, DPV 6.7 (20 points)

A subsequence is palindromic if it is the same whether read left to right or right to left. For instance, the sequence $A, C, G, T, G, T, C, A, A, A, A, T, C, G$ has many palindromic subsequences, including A, C, G, C, A and A, A, A, A (on the other hand, the subsequence A, C, T is not palindromic). Devise an $O(n^2)$-time algorithm that takes a sequence $x[1 \dots n]$ and returns the (length of the) longest palindromic subsequence.

5 Minimum Weight Triangulation (20 points)

Let P be a convex polygon with n vertices. A line segment connecting any two vertices u and v of P lies completely inside P; the weight of uv, denoted by w_{uv}, is the Euclidian distance between the vertices u and v. A triangulation of P polygon is a decomposition of the polygon into $n-2$ triangles. (See figures below). Notice that there are several ways of triangulating P. We define the weight of a triangulation of P to be the sum of the weights of its chords. Describe an $O(n^3)$-time algorithm to compute a minimum-weight triangulation of P.

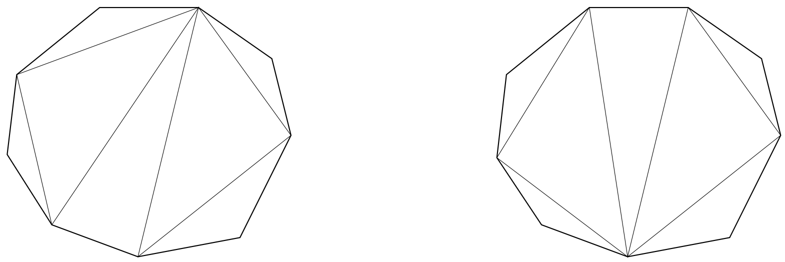

Figure 1: Two triangulations of a convex polygon.