Due Date: Thursday, Feb 16, 2012

1 Sorting [DPV 2.17] (20 points)

Given a sorted array of distinct integers $A[1...n]$, describe an $O(\log n)$-time algorithm to determine whether there is an index i such that $A[i] = i$.

2 Merging sorted arrays [DPV 2.22] (20 points)

Given two sorted lists of size m and n and an integer $1 \leq k \leq m+n$, describe an $O(\log m + \log n)$ time algorithm for computing the kth smallest element in the union of two lists.

3 Finding the majority element [DPV 2.23] (20 points)

An array $A[1...n]$ is said to have a majority element if more than half of its entries are same. Given an array, task is to design an efficient algorithm to tell whether array has a majority element, and if so, find the element. The elements of the array are not necessarily from some ordered domain, so only allowed operation is query of the form $A[i] = A[j]$.

- Show how to solve this problem in $O(n \log n)$ time.
  (**Hint:** *Divide the array into two smaller arrays. Does knowing the majority element of them help to figure out the majority element of A?*)

- Give a linear time algorithm for the same problem.
  (**Hint:** *Here is another approach. Pair up the elements of array to get $\frac{n}{2}$ pairs. In each pair, if elements are different discard both of them. If they are same, then keep one of them. Show that after this procedure, there are at most $\frac{n}{2}$ elements left and they have a majority element*)

4 Bipartite graphs [DPV 3.7] (20 points)

A bipartite graph is a graph $G = (V, E)$ whose vertices can be partitioned into two sets ($V = V_1 \cup V_2$) and $V_1 \cap V_2 = \varnothing$ such that there are no edges between vertices in the same set.

- Give a linear-time algorithm to dertermine whether an undirected graph is bipartite.

- Prove that an undirected graph is bipartite if and only if it contains no cycles of odd length.

5 Finding Cycles [DPV 3.11] (20 points)

Design a linear-time algorithm which, given an undirected graph G and an particular edge e in it, determines whether G has a cycle containing e.