

Intro to Game Theory

CPS 170

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(with many slides courtesy of Vince Conitzer)

What is game theory?

- Study of settings where multiple parties (**agents**) each have
 - different preferences (utility functions),
 - different actions
- Each agent's utility (potentially) depends on all agents' actions
 - What is optimal for one agent depends on what other agents do
 - Can be circular
- Game theory studies how agents can rationally form **beliefs** over what other agents will do, and (hence) how agents should **act**
- Can be useful for acting as well as (potentially) predicting behavior of others
- Not necessarily descriptive

Real World Game Theory Examples







- War
- Auctions
- Animal behavior
- Networking protocols, peer to peer networking behavior
- Road traffic
- Mechanism design: Suppose we want people to do X? How do we engineer the situation so that they will act that way?

Rock-paper-scissors

Column player AKA
player 2 (simultaneously)
chooses a column

Row player AKA
player 1 chooses
a row

A row or column is called
an **action** or (**pure**)
strategy

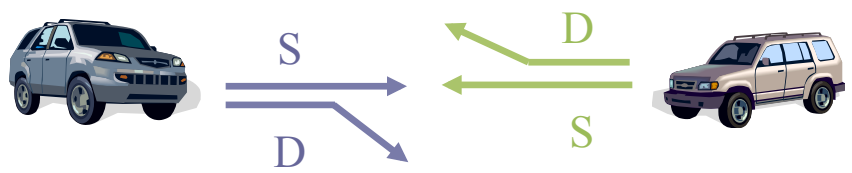
0, 0	-1, 1	1, -1
1, -1	0, 0	-1, 1
-1, 1	1, -1	0, 0

Row player's utility is always listed first, column player's second

Zero-sum game: the utilities in each entry sum to 0 (or a constant)
Three-player game would be a 3D table with 3 utilities per entry, etc.

“Chicken”

- Two players drive cars towards each other
- If one player goes straight, that player wins
- If both go straight, they both die



	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

not zero-sum

Rock-paper-scissors – Seinfeld variant



MICKEY: All right, rock beats paper!
 (Mickey smacks Kramer's hand for losing)
 KRAMER: I thought paper covered rock.
 MICKEY: Nah, rock flies right through paper.
 KRAMER: What beats rock?
 MICKEY: (looks at hand) Nothing beats rock.



	Rock	Paper	Scissors
Rock	0, 0	1, -1	1, -1
Paper	-1, 1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

Dominance

- Player i 's strategy s_i **strictly dominates** s_i' if
 - for any s_{-i} , $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$
- s_i **weakly dominates** s_i' if
 - for any s_{-i} , $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$; and
 - for some s_{-i} , $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$

$-i$ = "the player(s) other than i "

	0, 0	1, -1	1, -1
	-1, 1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

Prisoner's Dilemma

- Pair of criminals has been caught
- District attorney has evidence to convict them of a minor crime (1 year in jail); knows that they committed a major crime together (3 years in jail) but cannot prove it
- Offers them a deal:
 - If both confess to the major crime, they each get a 1 year reduction
 - If only one confesses, that one gets 3 years reduction

	confess	don't confess
confess	-2, -2	0, -3
don't confess	-3, 0	-1, -1

“Should I buy an SUV?”

purchasing + gas cost



cost: 5



cost: 3

accident cost



cost: 5

cost: 5



cost: 8


cost: 2



cost: 5

cost: 5



	-10, -10	-7, -11
	-11, -7	-8, -8

“2/3 of the average” game

- Everyone writes down a number between 0 and 100
- Person closest to 2/3 of the average wins
- Example:
 - A says 50
 - B says 10
 - C says 90
 - $\text{Average}(50, 10, 90) = 50$
 - $2/3 \text{ of average} = 33.33$
 - A is closest ($|50 - 33.33| = 16.67$), so A wins

Iterated dominance

- Iterated dominance: remove (strictly/weakly) dominated strategy, repeat
- Iterated strict dominance on Seinfeld's RPS:

	0, 0	1, -1	1, -1
	-1, 1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

→

	0, 0	1, -1
	-1, 1	0, 0

Mixed strategies

- **Mixed strategy** for player i = probability distribution over player i 's (pure) strategies
- E.g. $1/3$ $1/3$, $1/3$
- Example of dominance by a mixed strategy:

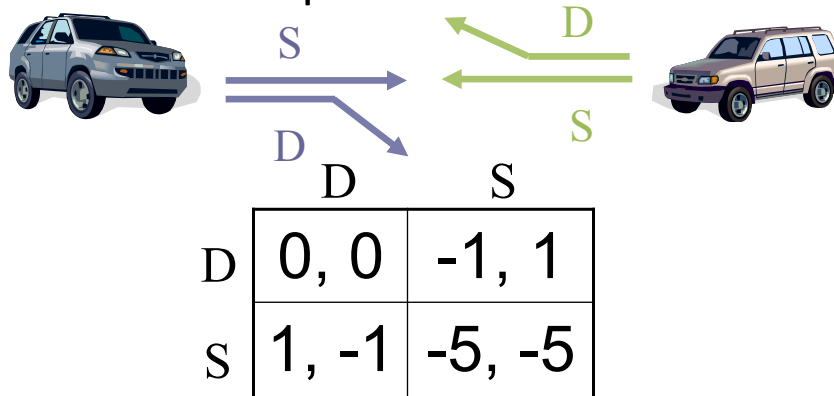
$1/2$	3, 0	0, 0
$1/2$	0, 0	3, 0
	1, 0	1, 0

Nash equilibrium [Nash 50]









- A vector of strategies (one for each player) = a **strategy profile**
- Strategy profile $(\sigma_1, \sigma_2, \dots, \sigma_n)$ is a **Nash equilibrium** if each σ_i is a **best response** to σ_{-i}
 - That is, for any i , for any σ'_i , $u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma'_i, \sigma_{-i})$
- Does not say anything about multiple agents changing their strategies at the same time
- In any (finite) game, at least one Nash equilibrium (possibly using mixed strategies) exists [Nash 50]
- (Note - singular: equilibrium, plural: equilibria)

Nash equilibria of “chicken”



- (D, S) and (S, D) are Nash equilibria
 - They are **pure-strategy Nash equilibria**: nobody randomizes
 - They are also **strict Nash equilibria**: changing your strategy will make you strictly worse off
- No other pure-strategy Nash equilibria

Rock-paper-scissors

			
	0, 0	-1, 1	1, -1
	1, -1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

- Any pure-strategy Nash equilibria?
- But it has a **mixed-strategy Nash equilibrium**:
Both players put probability 1/3 on each action

Nash equilibria of “chicken”...

	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

- Is there a Nash equilibrium that uses mixed strategies -- say, where player 1 uses a mixed strategy?
- If a mixed strategy is a best response, then all of the pure strategies that it randomizes over must also be best responses
- So we need to make player 1 **indifferent** between D and S
- Player 1's utility for playing D = $-p_S^c$
- Player 1's utility for playing S = $p_D^c - 5p_S^c = 1 - 6p_S^c$
- So we need $-p_S^c = 1 - 6p_S^c$ which means $p_S^c = 1/5$
- Then, player 2 needs to be indifferent as well
- Mixed-strategy Nash equilibrium: $((4/5 D, 1/5 S), (4/5 D, 1/5 S))$
 - People may die! Expected utility -1/5 for each player

$-p_S^c$ = probability
that column
player plays s

Computational Issues

- Zero-sum games can be solved efficiently as linear programs (see slides from earlier in the semester)
- General sum games may require exponential time (in # of actions) to find a single equilibrium (no known efficient algorithm and good reasons to suspect that none exists)
- Some better news: Despite bad worst-case complexity, many games can be solved quickly

Game Theory Issues

- How descriptive is game theory?
 - Some evidence that people play equilibria
 - Also, some evidence that people act irrationally
 - If it is computationally intractable to solve for equilibria of large games, seems unlikely that people are doing this
- How reasonable is game theory?
 - Are payoffs known?
 - Are situations really simultaneous move with no information about how the other player will act?
 - Are situations really single-shot

Extensions

- Partial information
- Uncertainty about the game parameters, e.g., payoffs (Bayesian games)
- Multistep games with distributions over next states (game theory + MDPs = stochastic games)
- Multistep + partial information (Partially observable stochastic games)
- Game theory is so general, that it can encompass essentially all aspects of strategic, multiagent behavior, e.g., negotiating, threats, bluffs, coalitions, bribes, etc.