Brief Comments on Game Theory

Ron Parr CPS 170

What is Game Theory

- Very general mathematical framework to study situations where multiple agents interact, including:
 - Popular notions of games
 - Everything up to and including multistep, multiagent, simultaneous move, partial information games
 - Can even including negotiating, posturing and uncertainty about the players and game itself
- von Neumann and Morgenstern (1944) was a major launching point for modern game theory
- Nash: Existence of equilibria in general sum games

Covered Today

- 2 player, zero sum simultaneous move games
- Example: Rock, Paper, Scissors
- Linear programming solution

Linear Programs (max formulation)

maximize: $c^T x$

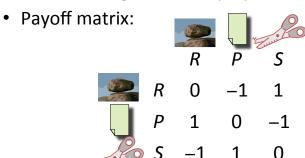
subject to: $\mathbf{A}x \le b$

 $: x \ge 0$

- Note: min formulation also possible
 - Min: c^Tx
 - Subject to: Ax≥b
- Some use equality as the canonical representation (introducing slack variables)
- LP tricks
 - Multiply by -1 to reverse inequalities
 - Can easily introduce equality constraints, or arbitrary domain constraints

Rock, Paper, Scissors Zero Sum Formulation

• In zero sum games, one player's loss is other's gain



• Minimax solution maximizes worst case outcome

Rock, Paper, Scissors Equations

- R,P,S = probability that we play rock, paper, or scissors respectively (R+P+S = 1)
- U is our expected utility
- Bounding our utility:
 - Opponent rock case: $U \le P S$
 - Opponent paper case: $U \le S R$
 - Opponent scissors case: U ≤ R P
- Want to maximize U subject to constraints
- Solution: (1/3, 1/3, 1/3)

Rock, Paper, Scissors LP Formulation

- Our variables are: x=[U,R,P,S]^T
- We want:
 - Maximize U
 - $-U \le P S$
 - $-U \leq S R$
 - $-U \leq R P$
 - -R+P+S=1
- How do we make this fit: subject to: $Ax \le b$

maximize: $c^T x$

 $: x \ge 0$

Rock Paper Scissors LP Formulation

$$X = \begin{bmatrix} U, R, P, S \end{bmatrix}^{T}$$

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 \end{bmatrix}$$

 $b = [0,0,0,1,-1]^T$

 $c = [1,0,0,0]^T$

maximize: $c^T x$

subject to: $\mathbf{A}x \le b$

 $: x \ge 0$

Rock, Paper, Scissors Solution

- If we feed this LP to an LP solver we get:
 - R=P=S=1/3
 - U=0
- Solution for the other player is:
 - The same...
 - By symmetry
- This is the minimax solution
- This is also an equilibrium
 - No player has an incentive to deviate
 - (Defined more precisely later in the course)

Tangent: Why is RPS Fun?

- OK, it's not...
- Why might RPS be fun?
 - Try to exploit non-randomness in your friends
 - Try to be random yourself

Minimax Solutions in General

- Minimax solutions for 2-player zero-sum games can always be found by solving a linear program
- The minimax solutions will also be equilibria
- For general sum games:
 - Minimax does not apply
 - Equilibria may not be unique
 - Need to search for equilibria using more computationally intensive methods