

Due Date: April 25, 2013, 11:59 PM

Problem 1: Polynomial Multiplication of FFT:

Suppose that you want to multiply the two polynomials $1+x+2x^2$ and $2+3x$ using the FFT. Choose an appropriate power of two, find the FFT of the two sequences, multiply the results componentwise, and compute the inverse FFT to get the final result.

Problem 2: Let X and Y be two sets of natural numbers such that the largest number in $X \cup Y$ is M . Let $Z = \{x+y \mid x \in X, y \in Y\}$ be the Minkowski sum of X and Y . Describe an $O(M \log M)$ algorithm to compute the set Z . ((**Hint:** Write each X and Y as coefficients of a polynomial.))

Problem 3: Consider a Boolean formula $\Phi(x_1, \dots, x_n)$ with clauses C_1, \dots, C_m . Φ is called *monotone* if each clause consists of only non-negated variables, i.e., no literal is of the form \bar{x}_i . For example,

$$\Phi(x_1, \dots, x_4) = (x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_4) \wedge (x_2 \vee x_4).$$

Consider the following problem: Given a monotone Boolean formula and an integer $k > 0$, is there a satisfiable assignment of Φ so that at most k variables are set to 1. (Note that Φ is obviously satisfiable if all variables are set to 1.) Show that this problem is NP-Complete.

Problem 4: Given an unweighted, directed graph $G = (V; E)$, a path (v_1, v_2, \dots, v_n) is a set of vertices such that for all $0 < i < n$, there is an edge from v_i to v_{i+1} . A cycle is a path such that there is also an edge from v_n to v_1 . A simple path is a path with no repeated vertices and, similarly, a simple cycle is a cycle with no repeated vertices. In this question we consider two problems:

LONGESTSIMPLEPATH: Given a graph $G = (V; E)$ and two vertices $u, v \in V$, find a simple path of maximum length from u to v or output NONE if no path exists.

LONGESTSIMPLECYCLE: Given a graph $G = (V; E)$, find a simple cycle of maximum length in G .

Reduce the problem of finding the longest simple path to the problem of finding the longest simple cycle. Prove the correctness of your reduction and show that it runs in polynomial time in V and E .

Problem 5: A film producer is seeking actors and investors for his new movie. There are n available actors; actor i charges s_i dollars. For funding, there are m available investors. Investor j will provide p_j dollars, but only on the condition that certain actors $L_j \subset 1, 2, \dots, n$ are included in the cast (all of these actors L_j must be chosen in order to receive funding from investor j). The producer's profit is the sum of the payments from investors minus the payments to actors. The goal is to maximize this profit. (a) Express this problem as an integer linear program in which the variables take on values 0, 1. (b) Now relax this to a linear program, and show that there must in fact be an integral optimal solution (as is the case, for example, with maximum flow and bipartite matching).

Problem 6: An integer program is a linear program with the additional constraint that the variables must take only integer values. Prove that deciding whether an integer program has a feasible solution is NP-hard. (**Hint:** *Almost any NP-hard decision problem can be formulated as an integer program. Take your pick.*)