Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

![DFA Diagram](image)

Turing Machine:

![Turing Machine Diagram](image)
Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM M is defined by $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ where

- $Q$ is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $B \in \Gamma$ is blank
- $q_0$ is start state
- $F$ is set of final states
- $\delta$ is transition function

$\delta(q,a) = (p,b,R)$ means “if in state q with the tape head pointing to an ’a’, then move into state p, write a ’b’ on the tape and move to the right”.
TM as Language recognizer

Definition: Configuration is denoted by $\vdash$.

if $\delta(q,a) = (p,b,R)$ then a move is denoted

$$abaqabba \vdash ababpbba$$

Definition: Let $M$ be a TM, $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$. $L(M) = \{ w \in \Sigma^* | q_0w^* \vdash x_1q_fx_2$ for some $q_f \in F$, $x_1, x_2 \in \Gamma^* \}$

TM as language acceptor

$M$ is a TM, $w$ is in $\Sigma^*$,

- if $w\in L(M)$ then $M$ halts in final state
- if $w \notin L(M)$ then either
  - $M$ halts in non-final state
  - $M$ doesn’t halt

Example

$\Sigma = \{a, b\}$

Replace every second ’a’ by a ’b’ if string is even length.

- Algorithm
Example:

\[ L = \{a^n b^n c^n | n \geq 1\} \]

Is the following TM Correct?

![TM Diagram]

**TM as a transducer**

TM can implement a function: \( f(w) = w' \)

- **start with:** \( w \)
- **end with:** \( w' \)
**Definition:** A function with domain $D$ is *Turing-computable* or *computable* if there exists TM $M=(Q,Σ,Γ,δ,q_0,B,F)$ such that

$$q_0w \xrightarrow{*} q_f f(w)$$

$q_f \in F$, for all $w \in D$.

**Example:**

$f(x) = 2x$

$x$ is a unary number

```
<table>
<thead>
<tr>
<th>Start with:</th>
<th>111</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>End with:</th>
<th>111111</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td></td>
</tr>
</tbody>
</table>
```

Is the following TM correct?

```
1;x,R
1;1,R
1;1,L

B;y,L
x;x,R

y;y,R

y;y,R

B;B,L
x;1;L

y;1,L

B;B,R

B;B,R

1;x,R

1;1,R

1;1,L

B;y,L
```
Example:

$L = \{ww \mid w \in \Sigma^+\}$, $\Sigma = \{a, b\}$