Proper Scoring Rules & Peer Prediction

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Overview

1. Proper Scoring Rules
   - Bonus: Prediction Polls

2. Classical Peer Prediction

CS-Econ Seminar (another bonus): Robust Peer Prediction
Proper Scoring Rules
Proper Scoring Rules [Brier, 1950]

Truthfully elicit beliefs about publicly observable events.

1. Agent reports belief $y \in [0, 1]$ of event occurring.

2. March 1: pay $R(y, \omega)$, where $\omega = \begin{cases} 1, & \text{if event occurs} \\ 0, & \text{if not.} \end{cases}$
Naive Approach: Linear Scoring Rule

Linear Scoring Rule

\[ R_l(y, \omega) = \begin{cases} 
  y, & \text{if } \omega = 1 \\
  1 - y, & \text{if } \omega = 0 
\end{cases} \]

True belief \( p = 0.6 \).

Expected score:
\[
0.6 \cdot y + 0.4 \cdot (1 - y) = 0.4 + 0.2y
\]

\[ \Rightarrow y = 1 \neq p \text{ maximizes expected score.} \]

Linear Rule not proper!
Quadratic Scoring Rule

Quadratic Scoring Rule

\[ R_q(y, \omega) = 1 - (y - \omega)^2 \]

True belief: \( p = 0.6 \).

Expected score:

\[
p \cdot R_q(y, 1) + (1 - p) \cdot R_q(y, 0)
= 0.6 \cdot (1 - (y - 1)^2) + 0.4 \cdot (1 - (y - 0)^2)
= -y^2 + 1.2y + 0.4
\]

Derive and set to 0:

\[-2y + 1.2 := 0 \Leftrightarrow y = 0.6\]

Quadratic Rule is proper: \( y = p \) maximizes expected score!
Bonus: Prediction Polls™
Good Judgment Project (GJP)

- Forecasting tournament for geo-political questions.
- ~10,000 active forecasters.
- ~140 questions / year.
- Prediction markets and proper scoring rules.
- (Mostly) play money (leaderboard).

Probability elicitation in real world:

- Many forecasters: aggregation?
- Not one-shot: beliefs are continuously updated.
- Not every forecaster reports on every question.
- Not every question has same duration.

How do you translate Proper Scoring Rules into the real world?
Forecast Aggregation in GJP Prediction Polls

1. Take weighted average:
   - Current score (closed questions): previous accuracy.
   - Frequency of updates: ~effort.
   - Only $k$ most recent forecasts: robustness vs novelty.

2. Extremize:
   - If average < 0.5 $\Rightarrow$ push towards 0.
   - If average > 0.5 $\Rightarrow$ push towards 1.
Extremizing: Intuition

Probability of Heads (H) for biased $\text{̄}$?

- Before observing flip: $p(H) = 0.5$
- Two forecasters observe flip and report:
  - $p_1(H) = 0.7$
  - $p_2(H) = 0.7$
- Aggregated forecast:
  - Same coin flip $\Rightarrow p_{1,2}(H) = 0.7$
  - Same coin, different flips $\Rightarrow p_{1,2}(H) > 0.7$

Less information overlap $\Rightarrow$ more extremizing!
Peer Prediction
Motivation: Information Elicitation

Hotel review

**Radisson Austin Hotel**
Austin, Texas on Jan 25, 2015 to Jan 27, 2015

⭐⭐⭐
Amenities: Smoke Free Rooms, Fitness Center, Pool(s), Restaurant(s), Business Center, Laundry Facilities (self-service), High-Speed Internet Access
View amenity descriptions

Dear JENS,

We hope you enjoyed your stay at the **Radisson Austin Hotel** on Jan 25, 2015 to Jan 27, 2015. We take pride in providing you with the best experience possible.

Please take a few minutes to answer some questions about your recent hotel stay. We use your feedback to help us evaluate each hotel, as well as our own performance. Now, you can even write reviews.

Would you recommend this hotel to others?

- **Yes**
- **No**
- **Unsure**
Motivation: Information Elicitation

The Good Judgment Project™

#1244 Will India and/or Brazil become a permanent member of the U.N. Security Council before March 1?

A: Yes
B: No

Comment ID: 225814, assigned: 09/01/14

Unless the Presidential winner is resolved quickly, there is no chance the Security Agreement will be signed in Wales at the September 4-5 NATO meeting. Audit results are now delayed until at least Sep. 10 and both sides have pulled their observers out of the audit process. Searches for a unity government agreement have also gone nowhere and are unlikely to be successful anyhow. Chances of signing before November 1 seem low. Efforts are under way to find a way to keep American forces in Afghanistan beyond year-end without a security agreement and that would seems to rapidly be becoming the only option short of pulling out troops.

How useful is this comment?
- 1. Not at all Useful (No use of CHAMPS KNOW)
- 2. Slightly Useful
- 3. Useful
- 4. Very Useful
- 5. Extremely Useful (Great Integration of CHAMPS KNOW)
Motivation: Information Elicitation

Does this Blog Have Any Offensive Content?

Prohibited Sexual Material or Nudity:

- sexually explicit or overtly suggestive content
- nudity (frontal, back or side)
- nudity (particularly of the genitals) covered by a towel, hat or other means
- grabbing, holding or touching genitals or genital area
- transparent/sheer or wet material below the waist or covering women's nipples/breasts
- erections or outline of genitals through clothing
- bare skin one inch directly above the pubic area
- shirtless body shots indoors. shirtless body shots are only allowed in natural settings (e.g. beach or swimming pool)
- cleavage shots without a face
- pubic hair
- underwear, including underwear waistband showing above pants
- body/torso shots without a head/face

CLICK HERE! to visit the blog.

Did you find content on the website that is deemed offensive based on the criteria provided above?

- Yes
- No
Research Questions

1. How can opinions or experiences be elicited truthfully?

2. How can we incentivize effort for information acquisition?
Basic Setup

- Elicit informative signal (e.g. “high” or “low” experience).
- Ground truth never observed (e.g. true quality of hotel).
- Allow for payments.

Agents experience same environment:

Key assumption: signals are correlated!
Belief Model (Common Knowledge)

Agent $i$'s belief that agent $j$ observes $h$:

$$p(h|l) = 0.18$$
$$p(h|h) = 0.46$$
Minority Opinions

Agent $i$’s belief that agent $j$ observes $h$:

\[ p(h|l) = 0.18 \]
\[ p(h|h) = 0.46 \quad \leftarrow \text{minority opinion: } p(h|h) < p(l|h) \]

Is Chicago capital of Illinois? [Prelec and Seung, 2006]

People who know it’s not, still believe they’re in the minority.

Peer prediction mechanisms elicit minority opinions truthfully!
Output Agreement

Compare two agents’ reports and pay: \[
\begin{cases}
$2 & \text{if reports agree,} \\
$0 & \text{otherwise.}
\end{cases}
\]

Example with \( S_i = h \)

\[ E[\text{payment}] \text{ reporting } h: \$0.92 \quad E[\text{payment}] \text{ reporting } l: \$1.08 \]

Output Agreement not truthful!
Classical Peer Prediction [Miller et al., 2005]

Mechanism

\[ p(h|l) = 0.18 \]
\[ p(h|h) = 0.46 \]

Agents

Share same belief model.

Knows belief model.

Report: Signal.
Classical Peer Prediction: Mechanism

Classical Peer Prediction: Mechanism

\[ p(h|l) = 0.18 \]
\[ p(h|h) = 0.46 \]

“high” \( \Rightarrow \)

\[ R\left(0.46, \text{“low”}\right) \]

“low” \( \Rightarrow \)

Intuition

1. Define agent \( j \)’s signal report as event.
2. Restrict possible belief reports to possible posteriors.

Crucial: mechanism knows how to transform signal to belief!
Subsequent Work in Peer Prediction

- Linear Programming formulation [Jurca and Faltings, 2006]

- Collusion-inhibiting mechanisms [Jurca and Faltings, 2009]

- Multiple equilibria unavoidable
  [Waggoner and Chen, 2014]

- Mechanisms not needing to know belief model

- Mechanisms for subjective prior beliefs
  [W. and Parkes, 2012b, 2013]

- Effort incentives [W. et al, 2013]
Summary

2. Prediction Polls: aggregate forecasts in real-world system.
3. Peer Prediction: elicit opinions, experiences, or ratings.

CS-Econ: Peer Prediction with relaxed common knowledge!


A Bayesian Truth Serum for Subjective Data.

An algorithm that finds truth even if most people are wrong.

A Robust Bayesian Truth Serum for Non-binary Signals.

