1 Analysis

1. Determination of numerical integration weights. Consider a numerical quadrature that is accurate for all the functions in the space,

\[ T_N[−\pi,\pi] = \text{span}\{\cos(kx) : k \in \{0, 1, \cdots, N - 1\}, x \in [−\pi,\pi]\} \]

assuming equ–spaced sample locations.

(a) describe the sample locations ;
(b) describe the basis functions you choose to use ;
(c) describe the system of linear equations for determining the weights (the matrix and the right hand side).

2. Orthogonal projectors in an inner product space. Consider two familiar vector spaces : \(\mathbb{C}[a,b]\) and \(\mathbb{R}^n\), each is over the field \(\mathbb{R}\), \(n\) is potentially large. For each of the spaces, specify a proper subspace \(U\) of dimension \(k > 1\).

(a) Equip each of the vector space with a specific, well-defined inner product.
(b) Describe the orthogonal projectors \(P_U\) \(P_U^\perp\) that decouples any vector \(v\) into two orthogonal components, one is in the space \(U\), the other in \(U^\perp\).
(c) Design a bijection that is not necessarily preserving the vector length.

Optional. Describe a forward model, first in continuous form, of parallel-beam projections with a full angular scan, for the 2D case. Describe next the discrete counterpart. (Recall and polish Yilun’s idea)

2 Experiments

3. Pattern matching. Experiment with the provided codes and data. Observe whether or not the CC map changes with the following changes. Explain why it is or not invariant.

(a) The pixel values of the template are shifted or scaled by a constant, or both.
(b) When the template shape is not of rectangle shape, for example, the binary variable `reshape` is set to 1 in the data generation routine.
   If you identify a problem, give it a try to fix it.

4. Super-resolution. Experiment with the provided codes and data.

(a) Establish the forward model.
(b) Use the model to reconstruct a small image at fine resolution.
(c) Find an approach for large scale reconstruction.
(d) Evaluate the accuracy of the reconstruction, using the ground-truth image at fine resolution.