1 Analysis

1. Reformulation of Optimization Problems.
   Consider Linear programming (LP) and Quadratic programming (QP). For each, do the following respectively.

   (a) Find the standard form for the primal model with constraints.
   (b) Give the expression of the Lagrange objective and constraints.
   (c) Derive and describe the dual model.
       Decide whether or not the dual model is also of LP (QP).
   (d) Describe the KKT conditions for the LP problem, and describe the system of equations.
       Decide whether or not the KKT conditions are sufficient.

Optional.

   (e) Apply the same mechanism to a different class of optimization problem.
   (f) Explain why the dual model is always convex or concave (depending on the sign).
   (g) Describe the min-max saddle problem that embeds both the primal and dual models.
       Cite references if used any.

2. Find in the article, by Dahi and Hansen in 2010, titled *Algorithms and Software for Total Variations Image Reconstruction via First-order Methods*, the primal and dual models for one of the image processing problems: inpainting or denoising.

Optional. Get the models for both problems.

3. Describe first the difference in numerical solutions for solving a linear system and a non-linear system with Jacobi’s iteration.

Next, describe the difference between the Jacobi iteration and the Gauss-Seidel method.
2 Experiments

4. Consider numerical solution to the following small system of non-linear equations for the unknowns $(x, y)$

\[
\begin{align*}
    x^3 - 3x y^2 &= \alpha \\
    y^3 - 3x^2 y &= \beta
\end{align*}
\]

where $(\alpha, \beta)$ is a pair of real numbers in $(1/4, 4]$, after scaling.

Find or design an iterative method (with termination criteria) for the solution(s). When convergent, show the convergence rate experimentally. Show also the convergence map, divergence map, and the basins of attractions, within a reasonable domain of the initial values.

For data displays, provide all labels, titles, legends and set axis ranges properly.

5. Find a large, structure linear system of equations that you have encountered or are interested in. A linear system of about 500 unknowns is considered small on today’s computers/laptops.

Use an iterative method for the numerical solution. Experimentally describe the convergence rate, if convergent. Describe the use of subsystems, if applied. You may use a direct method to solve any subsystem of modest size.

6. Find a structure nonlinear system of equations (at least 5 unknowns) that you have encountered or are interested.

Use an iterative method for the numerical solution. Experimentally describe the convergence (and divergence) behaviors. Describe your subsystems if used any, and how the subsystem solutions are integrated with the whole system.