## Contents

1 Numerical sampling and weighting  
1.1 Forward model  
   1.1.1 A trivial case study  
   1.1.2 Exercise  
   1.1.3 A non-trivial case study  
   1.1.4 Basic issues
1 Numerical sampling and weighting

Prototype application: reconstruction from projection data

1.1 Forward model

In a forward model, we describe the data acquisition process and properties. Specifically, we assume the ground-truth data (in a continuum domain) and project the data to the sensor pixels (discrete and finite) following the acquisition process.

1.1.1 A trivial case study

See a toy example in Figure 1.

![Figure 1: Parallel projections: along rows, columns and anti-diagonals](image)

1.1.2 Exercise

1. Set the matrix. Relate the unknowns in $x$ on the $5 \times 5$ Cartesian grid to 19 measurements in $b$ obtained from three sets of parallel projections as shown in the standard form

\[ Ax = b \]

2. Make arguments for or against the claim or a modified version

\[ b \in \text{span}(A) \]

3. Make arguments for or against the claim that there exists a solution $x$.

4. Discuss on necessary and sufficient conditions for the uniqueness of the solution if exists.
1.1.3 A non-trivial case study

Now look at the case Figure 2\(^1\), which is closer to certain practical applications.

![Diagram](source.png)

Figure 2: Cone-beam projections at angle $\theta$

1.1.4 Basic issues

- Model discretization for each projection:
  - sampling: spatial discretization of each ray integration: equispaced or non-quispaced
  - weighting: zero-th order (proportional to associated spatial occupation) or higher order (target at certain representation basis functions as well)
  - design & analysis: numerical integration (efficiency & accuracy)

- Registration across projections
  - re-sampling: translate ray samples to a common set of grid points for reconstruction
  - (re-) weighting on reconstruction data points
  - design & analysis: numerical interpolation (efficiency & accuracy)

- Relate the unknowns on the reconstruction grid points to the sensor data.

$$Ax = b$$

where $x$ is on a Cartesian grid, $b$ contains sensor data from all projections taken.

- Determine solution conditions.

- Design data data structures for $x$, $b$ and $A$, which are large, for economic use of memory and efficient computation.

\(^1\)Courtesy of A. Iliopoulos.