COMPSCI 630
Randomized Algorithm

Spring 2016

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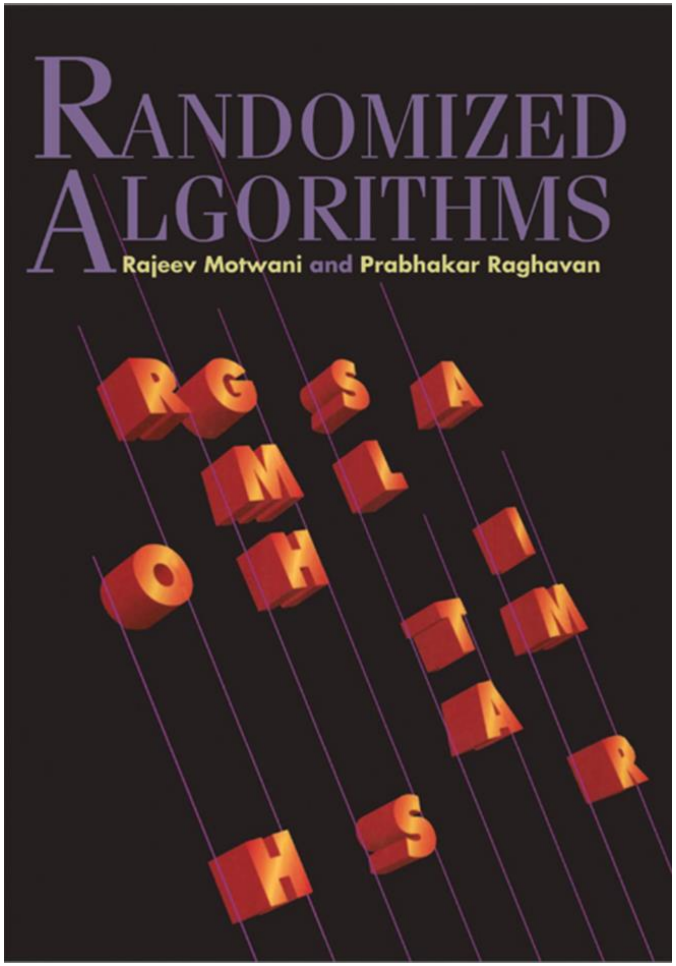
Basic Info

- Web Page: http://www.cs.duke.edu/courses/spring16/compsci630/
- Contact Me:
  LSRC D226
  Email: rongge@cs.duke.edu
- Office Hour:
  Wednesdays after class at 3:00 – 5:00 pm.

Basic Info

- Homework:
  3 or 4 problem sets, due 2 weeks after posted.
  Discussions are allowed, but *must acknowledge*.
- Piazza: Use Piazza for clarifications
  http://piazza.com/duke/spring2016/cps630
- Final Exam:
  There will be a take-home final for this course.
- Homework/Final will mostly involve proofs for correctness/running time of algorithms.

Textbook

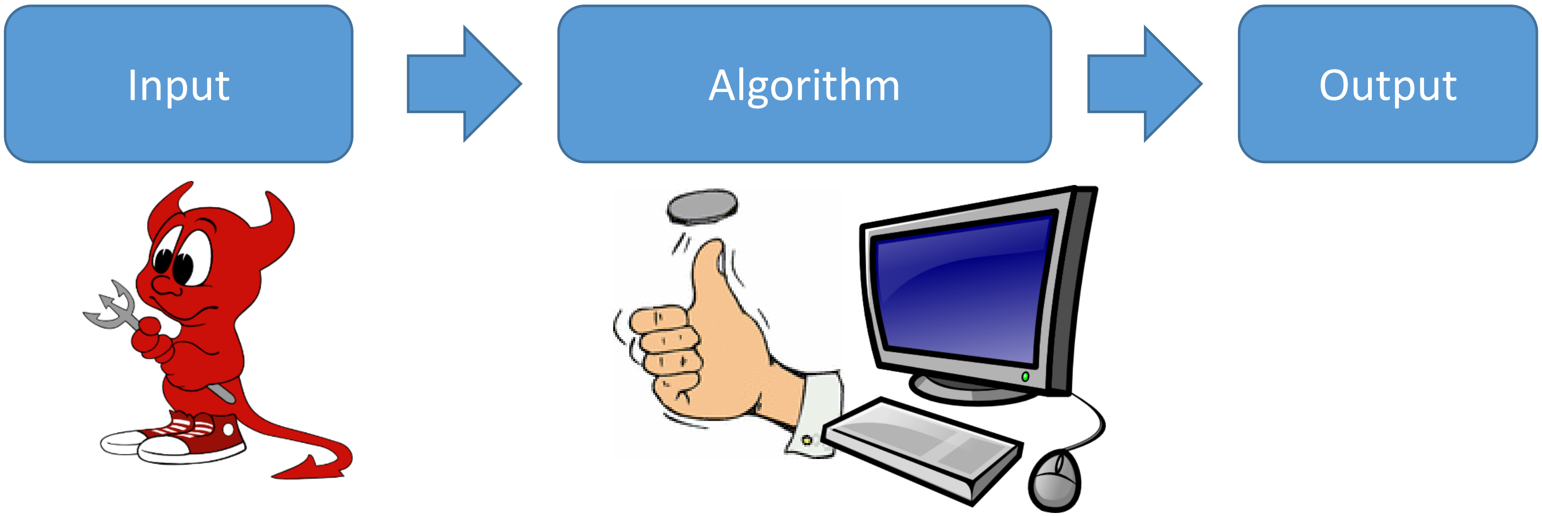

RANDOMIZED ALGORITHMS
Rajeev Motwani and Prabhakar Raghavan

Lecture 1: Overview

Outline

- What are randomized algorithms?
- Why do we need randomness?
- How do we analyze randomized algorithms?

Randomized Algorithm



- Input is still deterministic (worst-case analysis).
- Output is “correct” with probability at least 2/3 for every input.

Example 1

- Randomized quicksort

1	6	3	2	9	8	7	5	4	10

- Randomly pick a pivot, partition into two parts.

1	6	3	2	5	4

9	8	7	10

- Read intro in first Chapter for analysis

Example 2

- Find an number that's within top 50% in an array.

1	6	3	2	9	8	7	5	4	10

- Randomly pick two numbers, output the larger one

- Probability of Failure = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$!

Average Case Analysis

- Input is randomly generated (by nature).
- Output is "correct" for most inputs.

Average vs. Randomized Algorithm

- Average case is usually easier
  (If there is a randomized algorithm, there is also a good algorithm for any input distribution.)

- They are sometimes related (Min-Max Principal).

- Many tools apply to both settings.

Las Vegas Algorithm

- Always terminates with correct output.
- Running time is random, usually want to analyze the expected running time.

- Example: randomized quicksort

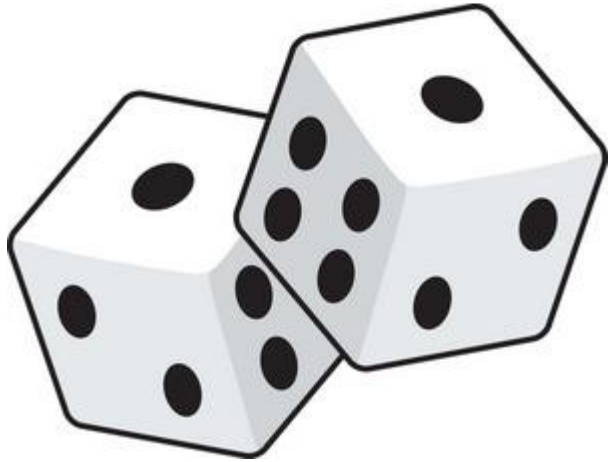
1	6	3	2	9	8	7	5	4	10

1	6	3	2	5	4

9	8	7	10

Monte Carlo Algorithm

- May produce incorrect outputs.
- Sometimes finish in fixed amount of time, sometimes can also have a random running time.
- Example: Finding a top 50% number in an array.
- Think: Can we transform between Las Vegas and Monte Carlo algorithms?

Why is randomness useful?

Randomness can…

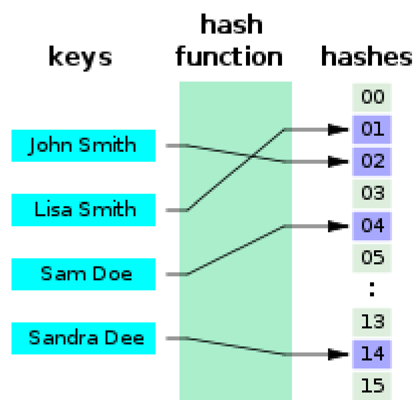
- Avoid Worst-Case Scenarios.
- Randomized Quicksort

1	6	3	2	9	8	7	5	4	10

1	6	3	2	5	4

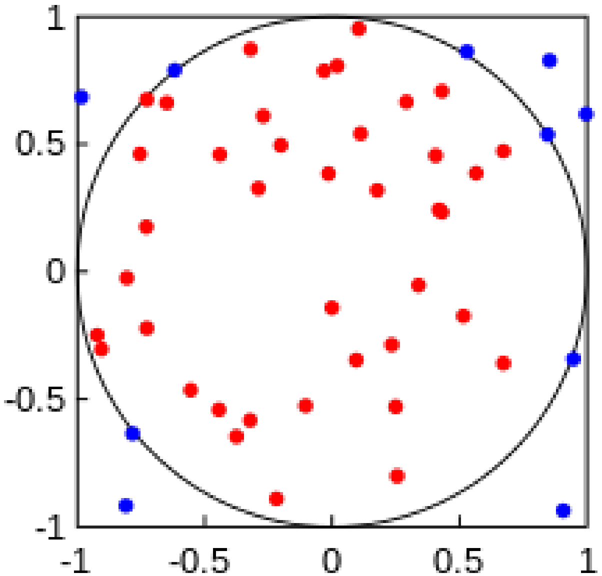
9	8	7	10

- Hashing



Randomness can…

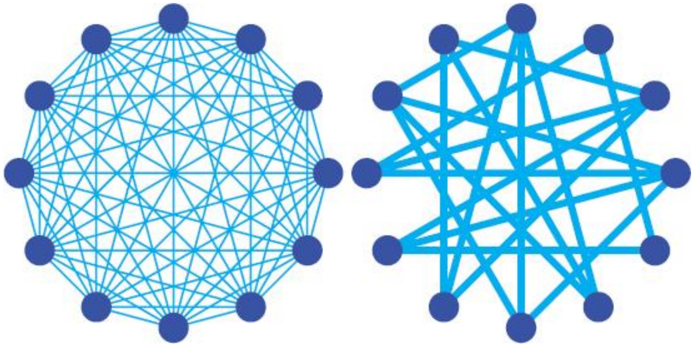
- Give a rough estimate very quickly.
- Monte-Carlo algorithm for computing volume.



Randomness can…

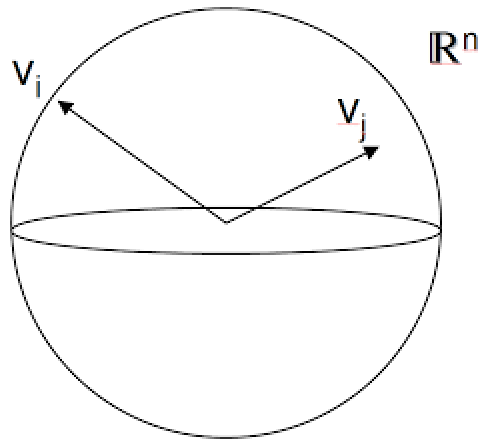
- Compress data while preserving crucial information.

- Dimension reduction
- Graph sparsification



Randomness can…

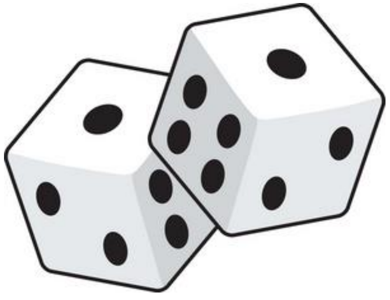
- Round Linear Program/Semidefinite Programs.



Tools to Analyze Randomized Algorithms

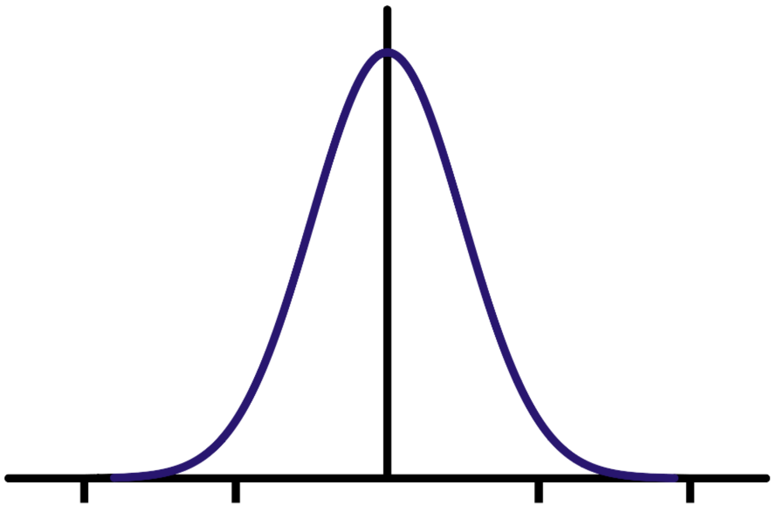
Tools

- Basic properties of random variables.
  - Linearity of expectation
  - Independence
  - Union bound
  - Markov/Chebyshev inequalities.
- Chapters 1, 3 in book.



Tools

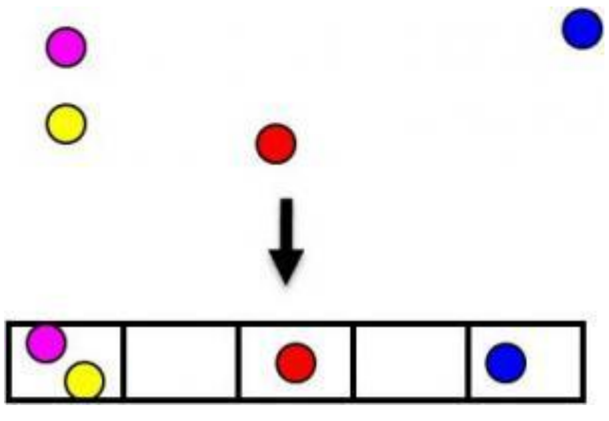
- Concentration Bounds, Martingales
- $f(X) \approx \mathbb{E}[f(X)]$
- Chapter 4 in book.



Tools

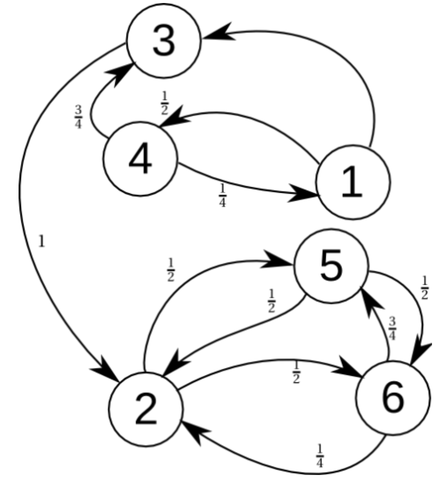
- Dimension Reduction

- Useful for fast algorithms with approximate solutions.
- We will see examples in many different settings.



Tools

- Markov Chains
- Powerful tool for sampling/counting.
- Chapters 6, 11 in book



Tools

- Complexity, derandomization.
- Is there a problem that has a polynomial time randomized algorithm but no deterministic polynomial time algorithm?

What’s not covered in detail

- Game theory
- Randomization is inherent in the solution space!
- CPS 590.4 Computational Microeconomics: Game Theory, Social Choice, and Mechanism Design

- Read Chapter 2 for a short intro.

What’s not covered in detail

- Online Algorithms
- Randomization is crucial in algorithms/analysis.
- CPS590 Optimization and Decision-making under Uncertainty

- Read Chapter 13 for a short intro.

What's not covered in detail

- Geometry Algorithms
- Many elegant applications of randomness.
- CPS634 Geometric Algorithms

- Read Chapter 10 for a few examples.