SQL: Index and Recursion

Introduction to Databases

CompSci 316 Spring 2017

DUKE COMPUTER SCIENCE

Announcements (Wed., Feb. 15)

- Homework #2 due Friday 02/17
  - Except Problem 6 (Gradiance) and Problem X1 (non-Gradiance) : Thursday 02/23
  - Please submit on time – solutions will be posted by Saturday morning
- Midterm next Wednesday 02/22 in class
  - Up to lecture 9 included
  - Will review some concepts/practice problems on Monday

Indexes

- An index is an auxiliary persistent data structure
  - Search tree (e.g., B$^+$-tree), lookup table (e.g., hash table), etc.
  - ☞More on indexes later in this course!
- An index on $R.A$ can speed up accesses of the form
  - $R.A = value$
  - $R.A > value$ (sometimes; depending on the index type)
- An index on $(R.A_1, \ldots, R.A_n)$ can speed up
  - $R.A_1 = value_1 \wedge \cdots \wedge R.A_n = value_n$
  - $(R.A_1, \ldots, R.A_n) > (value_1, \ldots, value_n)$ (again depends)

☞Ordering or index columns is important—is an index on $(R.A, R.B)$ equivalent to one on $(R.B, R.A)$?

☞How about an index on $R.A$ plus another on $R.B$?

Examples of using indexes: 1/2

- SELECT * FROM User WHERE name = 'Bart';
- Without an index on *User.name*:
- With an index on *User.name*:

Examples of using indexes: 2/2

- SELECT * FROM User, Member
  WHERE User.uid = Member.uid
  AND Member.gid = 'jes';

Recall the semantic for SQL evaluation!

- With an index on *Member.gid* or (*gid*, *uid*):
- With an index on *User.uid*:
- Without an index:

Creating and dropping indexes in SQL

CREATE [UNIQUE] INDEX *indexname* ON *tablename*($columnname_1$,…,$columnname_n$);

- With UNIQUE, the DBMS will also enforce that $\{columnname_1, \ldots, columnname_n\}$ is a key of *tablename*

DROP INDEX *indexname*;

- Typically, the DBMS will automatically create indexes for PRIMARY KEY and UNIQUE constraint declarations

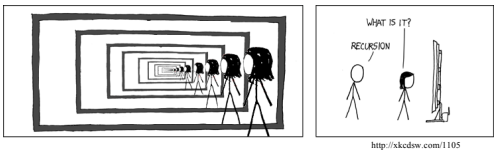
Choosing indexes to create

More indexes = better performance?

☞Optimal index selection depends on both query and update workload and the size of tables

- Automatic index selection is now featured in some commercial DBMS

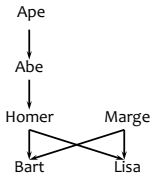
Next: Recursion!



A motivating example

Parent (*parent*, *child*)

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe



- Example: find Bart's ancestors
- "Ancestor" has a recursive definition
  - *X* is *Y*'s ancestor if
    - *X* is *Y*'s parent, or
    - *X* is *Z*'s ancestor and *Z* is *Y*'s ancestor

Recursion in SQL

- SQL2 had no recursion
  - You can find Bart's parents, grandparents, great grandparents, etc.

```
SELECT p1.parent AS grandparent
FROM Parent p1, Parent p2
WHERE p1.child = p2.parent
AND p2.child = 'Bart';
```

  - But you cannot find all his ancestors with a single query
- SQL3 introduces recursion
  - WITH clause
  - Implemented in PostgreSQL (common table expressions)

Ancestor query in SQL3

Define a relation recursively

```
WITH RECURSIVE
Ancestor(anc, desc) AS
(
    (SELECT parent, child FROM Parent)        -- base case
UNION
    (SELECT a1.anc, a2.desc                   -- recursion step
     FROM Ancestor a1, Ancestor a2
     WHERE a1.desc = a2.anc)
)
SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
```

Query using the relation defined in WITH clause

Fixed point of a function

- If $f: T \to T$ is a function from a type T to itself, a fixed point of f is a value x such that $f(x) = x$
- Example: What is the fixed point of $f(x) = x/2$?
  - 0, because $f(0) = 0/2 = 0$

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To compute fixed point of a function f

- Start with a "seed": $x \leftarrow x_0$
- Compute $f(x)$
  - If $f(x) = x$, stop; x is fixed point of f
  - Otherwise, $x \leftarrow f(x)$; repeat
- Example: compute the fixed point of $f(x) = x/2$
  - With seed 1: 1, 1/2, 1/4, 1/8, 1/16, … → 0

☞Doesn't always work, but happens to work for us!

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Fixed point of a query

- A query q is just a function that maps an input table to an output table, so a fixed point of q is a table T such that $q(T) = T$

To compute fixed point of q

- Start with an empty table: $T \leftarrow \emptyset$
- Evaluate q over T
  - If the result is identical to T, stop; T is a fixed point
  - Otherwise, let T be the new result; repeat

☞Starting from ∅ produces the unique minimal fixed point (assuming q is monotone)

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Finding ancestors

- WITH RECURSIVE Ancestor(anc, desc) AS ((SELECT parent, child FROM Parent) UNION (SELECT a1.anc, a2.desc FROM Ancestor a1, Ancestor a2 WHERE a1.desc = a2.anc))
  - Think of the definition as *Ancestor = q(Ancestor)*

parent	child
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	desc

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa

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Intuition behind fixed-point iteration

- Initially, we know nothing about ancestor-descendent relationships
- In the first step, we deduce that parents and children form ancestor-descendent relationships
- In each subsequent steps, we use the facts deduced in previous steps to get more ancestor-descendent relationships
- We stop when no new facts can be proven

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Linear recursion

- With linear recursion, a recursive definition can make only one reference to itself
- Non-linear
  - WITH RECURSIVE Ancestor(anc, desc) AS ((SELECT parent, child FROM Parent) UNION (SELECT a1.anc, a2.desc FROM Ancestor a1, Ancestor a2 WHERE a1.desc = a2.anc))
- Linear
  - WITH RECURSIVE Ancestor(anc, desc) AS ((SELECT parent, child FROM Parent) UNION (SELECT anc, child FROM Ancestor, Parent WHERE desc = parent))

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Linear vs. non-linear recursion

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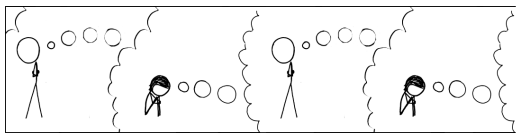

http://xkcdsw.com/3080

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Mutual recursion example

- Table *Natural* (*n*) contains 1, 2, … , 100

- Which numbers are even/odd?
  - An odd number plus 1 is an even number
  - An even number plus 1 is an odd number
  - 1 is an odd number

```
WITH RECURSIVE Even(n) AS
    (SELECT n FROM Natural
     WHERE n = ANY(SELECT n+1 FROM Odd)),
    RECURSIVE Odd(n) AS
    ((SELECT n FROM Natural WHERE n = 1)
     UNION
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Even)))
```

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Semantics of WITH

- WITH RECURSIVE R_1 AS Q_1, …,
  RECURSIVE R_n AS Q_n
  Q;
  - Q and $Q_1, \ldots, Q_n$ may refer to $R_1, \ldots, R_n$

- Semantics
  1. $R_1 \leftarrow \emptyset, \ldots, R_n \leftarrow \emptyset$
  2. Evaluate $Q_1, \ldots, Q_n$ using the current contents of $R_1, \ldots, R_n$:
     $R_1^{new} \leftarrow Q_1, \ldots, R_n^{new} \leftarrow Q_n$
  3. If $R_i^{new} \neq R_i$ for some i
     3.1. $R_1 \leftarrow R_1^{new}, \ldots, R_n \leftarrow R_n^{new}$
     3.2. Go to 2.
  4. Compute Q using the current contents of $R_1, \ldots R_n$ and output the result

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Computing mutual recursion

```
WITH RECURSIVE Even(n) AS
    (SELECT n FROM Natural
     WHERE n = ANY(SELECT n+1 FROM Odd)),
    RECURSIVE Odd(n) AS
    ((SELECT n FROM Natural WHERE n = 1)
     UNION
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Even)))
```

- *Even* = ∅, *Odd* = ∅
- *Even* = ∅, *Odd* = {1}
- *Even* = {2}, *Odd* = {1}
- *Even* = {2}, *Odd* = {1, 3}
- *Even* = {2, 4}, *Odd* = {1, 3}
- *Even* = {2, 4}, *Odd* = {1, 3, 5}
- …

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Fixed points are not unique

```
WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
```

parent	child
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Bogus	**Bogus**

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa
Bogus	**Bogus**

Note how the bogus tuple reinforces itself!

- But if q is monotone, then all these fixed points must contain the fixed point we computed from fixed-point iteration starting with ∅

- Thus the unique minimal fixed point is the "natural" answer

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Mixing negation with recursion

- If q is non-monotone
  - The fixed-point iteration may flip-flop and never converge
  - There could be multiple minimal fixed points—we wouldn't know which one to pick as answer!

- Example: popular users (pop ≥ 0.8) join either Jessica's Circle or Tommy's (but not both)
  - Those not in Jessica's Circle should be in Tom's
  - Those not in Tom's Circle should be in Jessica's

```
• WITH RECURSIVE TommyCircle(uid) AS
    (SELECT uid FROM User WHERE pop >= 0.8
     AND uid NOT IN (SELECT uid FROM JessicaCircle)),
    RECURSIVE JessicaCircle(uid) AS
    (SELECT uid FROM User WHERE pop >= 0.8
     AND uid NOT IN (SELECT uid FROM TommyCircle))
```

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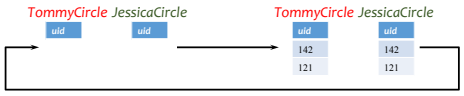
Fixed-point iter may not converge

- WITH RECURSIVE TommyCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
  AND uid NOT IN (SELECT uid FROM JessicaCircle)),
  RECURSIVE JessicaCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
  AND uid NOT IN (SELECT uid FROM TommyCircle))

uid	name	age	pop
142	Bart	10	0.9
121	Allison	8	0.85



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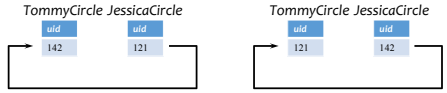
Multiple minimal fixed points

- WITH RECURSIVE TommyCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
  AND uid NOT IN (SELECT uid FROM JessicaCircle)),
  RECURSIVE JessicaCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
  AND uid NOT IN (SELECT uid FROM TommyCircle))

uid	name	age	pop
142	Bart	10	0.9
121	Allison	8	0.85



Problem: What do we answer if someone asks whether 121 belongs to JessicaCircle?

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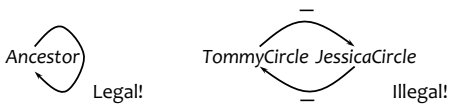
Legal mix of negation and recursion

- Construct a dependency graph
  - One node for each table defined in WITH
  - A directed edge $R \rightarrow S$ if R is defined in terms of S
  - Label the directed edge "–" if the query defining R is not monotone with respect to S
- Legal SQL3 recursion: no cycle with a "–" edge
  - Called stratified negation
- Bad mix: a cycle with at least one edge labeled "–"



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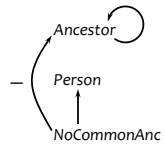
Stratified negation example

- Find pairs of persons with no common ancestors

```
WITH RECURSIVE Ancestor(anc, desc) AS
   ((SELECT parent, child FROM Parent) UNION
    (SELECT a1.anc, a2.desc
     FROM Ancestor a1, Ancestor a2
     WHERE a1.desc = a2.anc)),
   Person(person) AS
   ((SELECT parent FROM Parent) UNION
    (SELECT child FROM Parent)),
   NoCommonAnc(person1, person2) AS
   ((SELECT p1.person, p2.person
     FROM Person p1, Person p2
     WHERE p1.person <> p2.person)
    EXCEPT
    (SELECT a1.desc, a2.desc
     FROM Ancestor a1, Ancestor a2
     WHERE a1.anc = a2.anc))
SELECT * FROM NoCommonAnc;
```



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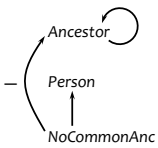
Evaluating stratified negation

- The stratum of a node R is the maximum number of "–" edges on any path from R in the dependency graph
  - *Ancestor*: stratum 0
  - *Person*: stratum 0
  - *NoCommonAnc*: stratum 1



- Evaluation strategy
  - Compute tables lowest-stratum first
  - For each stratum, use fixed-point iteration on all nodes in that stratum
    - Stratum 0: *Ancestor* and *Person*
    - Stratum 1: *NoCommonAnc*

☞Intuitively, there is no negation within each stratum

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Datalog: Another query language for recursion

- Ancestor(x, y) :- Parent(x, y)
- Ancestor(x, y):- Parent(x, z), Ancestor(z, y)

- Like logic programming
- Multiple rules
- Same "head" = union
- "," = AND

- Same semantics that we discussed so far
  - not covered in detail in this class

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Summary

- SQL3 WITH recursive queries
- Solution to a recursive query (with no negation): unique minimal fixed point
- Computing unique minimal fixed point: fixed-point iteration starting from ∅
- Mixing negation and recursion is tricky
  - Illegal mix: fixed-point iteration may not converge; there may be multiple minimal fixed points
  - Legal mix: stratified negation (compute by fixed-point iteration stratum by stratum)
- Another language for recursion: Datalog