COMPSCI630 Randomized Algorithms
Assignment 0

Due Date: Friday Jan. 19

Problem 1 (Karger-Stein Algorithm). We've talked about Karger-Stein algorithm for MIN-CUT in class:

Algorithm 1 Fast-Min-Cut(G)

```
if |V| ≤ 8 then
  Return Min-Cut(G)
else
  Let t = ⌈|V|/√2⌉.
  Let G1 = Contract(G, t); G2 = Contract(G, t);
  Return min{Fast-Min-Cut(G1), Fast-Min-Cut(G2)}.
end if
```

Here Min-Cut(G) is an algorithm for solving MIN CUT on small graphs; Contract(G, t) is similar to Karger's algorithm and will randomly contract edges in G until there are only t vertices left.

In this problem we would like to show the success probability of the algorithm is $\Omega(1/\log n)$ for a graph of n vertices.

(a) Let $P(n)$ be the probability of success for the algorithm on n vertices, express $P(n)$ recursively using $P(n/\sqrt{2})$. (Here for simplicity, you can assume $n/\sqrt{2}$ is an integer. Also, you can assume that the probability that the min cut of G_1 is equal to min cut of G is exactly $1/2$.)

(b) Let $f(t) = P(\sqrt{2}^t)$, use induction to prove that $f(t) \geq 4/t$ for $t \geq 4$.

Problem 2 (Distinct Min Cuts). (a) Prove that any graph G with n vertices can have at most $\binom{n}{2}$ distinct min cuts. (Two cuts are different if they contain different set of edges.)

(Hint: Look at the success probability of Karger's min cut algorithm.)

(b) For any n, construct a graph that has exactly $\binom{n}{2}$ distinct min cuts.