Due on February 4, 2019
81 points total

General directions: We will exclusively use Python 3 for our programming assignments, and allow only the use of modules in the Python 3 standard library unless explicitly specified otherwise on an individual assignment basis. This forbids the use of common third-party libraries such as Numpy, Sympy, etc., but not the use of math or io.

Unless specified otherwise, for the X-th homework, download the single "hwX_skeleton.py" file from the course website, and rename it to "hwX.py" on your machine. When you are done and ready to submit, upload your file named **exactly** "hwX.py" on Gradescope for assignment "HW X (Programming)." When you upload your file, the autograder will run a simple test for each function so that you can confirm it was properly uploaded and executed. Generally, if an assignment involves printing or writing a file in a specific format, there will be at least one simple test that checks your output is formatted as we expect. These tests are not worth any credit — once the due date is over, your submission will be graded by a collection of additional test cases.

All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a "+1" in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (10+1 points)
Using the WOP, prove that for any positive integer n, $3^{n+2} + 2^{4n+2}$ is divisible by 13.

Problem 2 (35+4 points)
Consider the following sets:

$$A = \left\{ \frac{1}{2} - \frac{1}{2^k} : k \text{ is a positive integer} \right\}$$
$$B = \left\{ \frac{1}{2} + \frac{1}{2^k} : k \text{ is a positive integer} \right\}.$$

(a) **(10+1 points)** Prove or disprove that A is well-ordered.

(b) **(10+1 points)** Prove or disprove that B is well-ordered.

(c) **(10+1 points)** Prove that A satisfies the following property: for any real number x such that $0 < x < \frac{1}{2}$ and any positive integer n, there exist n distinct elements of A that are all greater than x. In other words, A contains decreasing sequences that are arbitrarily long.

(d) **(5+1 points)** Does A contain an infinite decreasing sequence? Justify your answer.

Problem 3 (14+1 points)
Consider the operator $\oplus$ defined so that $P \oplus Q$ is TRUE if P and Q have different truth values, and FALSE otherwise. Prove or disprove each of the following.

(a) **(3 points)** $\oplus$ is associative.

(b) **(3 points)** $\oplus$ is commutative.

(c) **(3 points)** $\oplus$ is idempotent.

(d) **(5 points)** $P \oplus Q \leftrightarrow \neg(Q \rightarrow P) \vee (P \wedge \neg Q)$.

Problem 4 (15+1 points)
Consider the operator $\#$ defined so that $P\#Q$ is TRUE if and only if P is FALSE and Q is TRUE. For each of the following expressions, give an equivalent expression which uses only the $\#$ and $\neg$ operators. That is, your expression should have only P, Q, $\neg$, $\#$, and parentheses (where each can be used any number of times).

(a) **(5 points)** $P \wedge Q$.

(b) **(5 points)** $P \vee Q$.

(c) **(5 points)** $P \rightarrow Q$.