Due on April 8, 2019
39 points total

General directions: We will exclusively use Python 3 for our programming assignments, and allow only the use of modules in the Python 3 standard library unless explicitly specified otherwise on an individual assignment basis. This forbids the use of common third-party libraries such as Numpy, Sympy, etc., but not the use of math or io.

Unless specified otherwise, for the X-th homework, download the single “hwX_skeleton.py” file from the course website, and rename it to “hwX.py” on your machine. When you are done and ready to submit, upload your file named **exactly** “hwX.py” on Gradescope for assignment “HW X (Programming).” When you upload your file, the autograder will run a simple test for each function so that you can confirm it was properly uploaded and executed. Generally, if an assignment involves printing or writing a file in a specific format, there will be at least one simple test that checks your output is formatted as we expect. These tests are not worth any credit — once the due date is over, your submission will be graded by a collection of additional test cases.

All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment “HW X (PDF).” Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a ”+1” in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (15+1 points)
Let A be a countable set. Prove that the set of all finite subsets of A is countable.

Problem 2 (22+1 points)
Recall that the set of real numbers is uncountable. In this question, we will consider other uncountably infinite sets. First, let (a, b) be the open interval from a to b, i.e. $(a, b) = \{r \in \mathbb{R} : a < r < b\}$. Similarly, $[a, b]$ is the closed interval from a to b, i.e. $[a, b] = \{r \in \mathbb{R} : a \leq r \leq b\}$. We can define a bijection from $\mathbb{R}$ to any finite interval. For example, we can create a bijection $f : \mathbb{R} \to (0, 1)$ by letting $f(x) = \arctan(x)$ for $x \in \mathbb{R}$.

(a) *(3 points)* Let $a, b \in \mathbb{R}$ such that $a < b$. Prove that (a, b) bij $[a, b]$.

(b) *(7 points)* Prove that there is a bijection between any two intervals of finite length. In other words, prove that $[a, b]$ bij $[c, d]$ for any $a < b, c < d$, where $a, b, c, d \in \mathbb{R}$.
(**Hint:** *Try to define a bijection using a linear function.*)

(c) *(12 points)* Let C be the unit circle, $C = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\}$. Prove that $\mathbb{R}$ bij C.
(**Hint:** *Consider first defining a bijection with a finite interval, then use polar coordinates.*)