Relational Model and Algebra

Introduction to Databases

CompSci 316 Spring 2019

DUKE COMPUTER SCIENCE

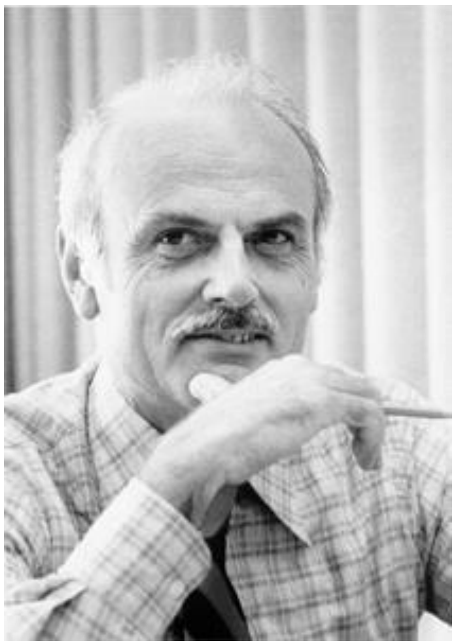
Announcements (Tue. Jan. 15)

- You should be on Piazza!
  - Otherwise, let me know after class
- HW1 to be posted today, due in about 2.5 weeks
  - Problems will be posted one by one after the material is covered in class (and announced on piazza)
  - Keep working on them
- Sign up for gradiance and gradescope
  - Tokens posted on Piazza

Announcements (Tue. Jan. 15)

- Set up VM
    - Instructions on course website
    - Google cloud coupon will be sent soon
    - There will be Help session next week
- TA/UTA office hours to be posted soon

Edgar F. Codd (1923-2003)



- Pilot in the Royal Air Force in WW2
- Inventor of the relational model and algebra while at IBM
- Turing Award, 1981

http://en.wikipedia.org/wiki/File:Edgar_F_Codd.jpg

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a set of attributes (or columns)
- Each attribute has a name and a domain (or type)
  - Set-valued attributes are not allowed
- Each relation contains a set of tuples (or rows)
  - Each tuple has a value for each attribute of the relation
  - Duplicate tuples are not allowed
    - Two tuples are duplicates if they agree on all attributes

☞Simplicity is a virtue!

Example

Attributes.

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
… 142	… Bard	… NULL	… 0.9

≠

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

Ordering of rows doesn't matter
(even though output is
always in some order)

Schema vs. instance

- Schema (metadata)
  - Specifies the logical structure of data
  - Is defined at setup time
  - Rarely changes

Group(gid, name)

- Instance
  - Represents the data content
  - Changes rapidly, but always conforms to the schema

gid	name
G1	Paul
G1	Alice

gid	name
G1	Paul
G2	Sarah

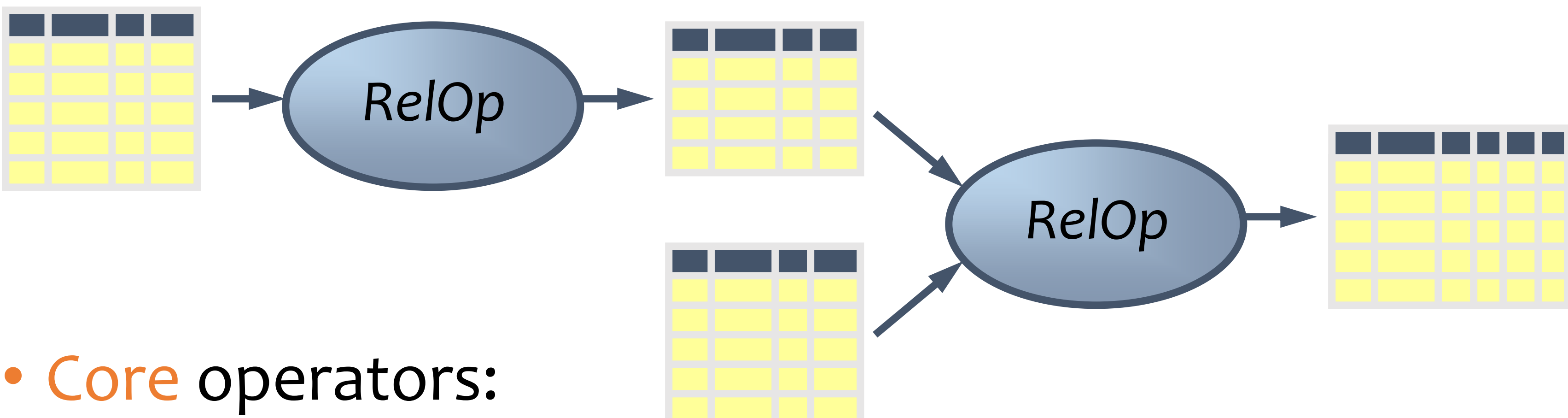
☞ Compare to types vs. collections of objects of these types in a programming language

Example

- Schema
  - *User* (*uid* int, *name* string, *age* int, *pop* float)
  - *Group* (*gid* string, *name* string)
  - *Member* (*uid* int, *gid* string)
- Instance
  - *User*: {⟨142, Bart, 10, 0.9⟩, ⟨857, Milhouse, 10, 0.2⟩, … }
  - *Group*: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, … }
  - *Member*: {⟨142, dps⟩, ⟨123, gov⟩, … }

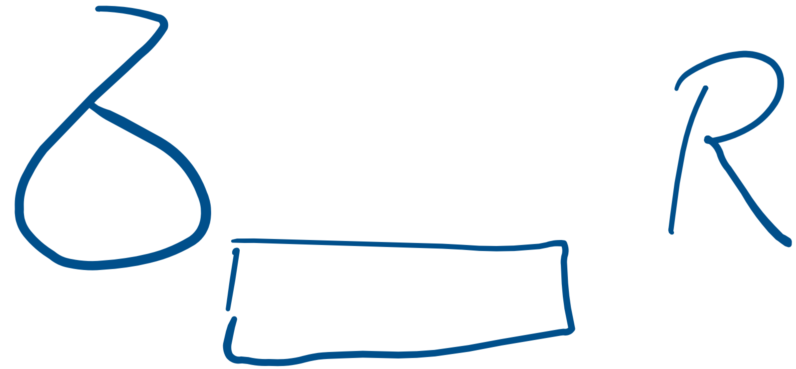
Relational algebra

A language for querying relational data based on "operators"



- Core operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

Selection



- Input: a table R
- Notation: $\sigma_p R$
  - p is called a selection condition (or predicate)
- Purpose: filter rows according to some criteria
- Output: same columns as R, but only rows or R that satisfy p

Selection example

- Users with popularity higher than 0.5

$$\sigma_{pop>0.5} User$$

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

$$\rightarrow \sigma_{pop>0.5} \rightarrow$$

uid	*name*	*age*	*pop*
142	Bart	10	0.9
[redacted]	[redacted]	[redacted]	[redacted]
857	Lisa	8	0.7
[redacted]	[redacted]	[redacted]	[redacted]
…	…	…	…

No actual deletion!

More on selection

- Selection condition can include any column of R, constants, comparison ($=$, $\leq$, etc.) and Boolean connectives (∧: and, ∨: or, ¬: not)
  - Example: users with popularity at least 0.9 and age under 10 or above 12

$$\sigma_{pop \geq 0.9 \wedge (age < 10 \vee age > 12)}\ User$$

- You must be able to evaluate the condition over each single row of the input table!
  - Example: the most popular user

$$\sigma_{pop \geq \text{every pop in } User}\ User$$

WRONG!

σ name like 'B%' User

Projection

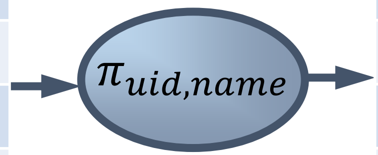
- Input: a table R
- Notation: $\pi_L R$
  - L is a list of columns in R
- Purpose: output chosen columns
- Output: same rows, but only the columns in L

Projection example

- IDs and names of all users

$$\pi_{uid,name}\ User$$

uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…



uid	name
142	Bart
123	Milhouse
857	Lisa
456	Ralph
…	…

More on projection

- Duplicate output rows are removed (by definition)
  - Example: user ages

$$\pi_{age}\, User$$

uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

→ π_{age} →

age
10
[redacted]
8
[redacted]
…

RA = set = no duplicate

distinct

(uid, ...) → Lisa

(lisa, 9)

Select age From User

10	
10	
8	
8	

Cross product

- Input: two tables R and S
- Natation: $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row r in R and each s in S, output a row rs (concatenation of r and s)

$$\{a, d, e\} \times \{1, 2\}$$
$$= \{(a,1), (d,1), (e,1), (a,2), (d,2), (e,2)\}$$

Cross product example

$$User \times Member$$

uid	*name*	*age*	*pop*
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	*gid*
123	gov
857	abc
857	gov
…	…

$\times$

uid	*name*	*age*	*pop*	*uid*	*gid*
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

A note a column ordering

- Ordering of columns is unimportant as far as contents are concerned

uid	*name*	*age*	*pop*	*uid*	*gid*
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

$=$

uid	*gid*	*uid*	*name*	*age*	*pop*
123	gov	123	Milhouse	10	0.2
857	abc	123	Milhouse	10	0.2
857	gov	123	Milhouse	10	0.2
123	gov	857	Lisa	8	0.7
857	abc	857	Lisa	8	0.7
857	gov	857	Lisa	8	0.7
…	…	…	…	…	…

- So cross product is commutative, i.e., for any R and S, $R \times S = S \times R$ (up to the ordering of columns)

Derived operator: join

(A.k.a. "theta-join")

- Input: two tables R and S
- Notation: $R \bowtie_p S$
  - p is called a join condition (or predicate)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row r in R and each row s in S, output a row rs if r and s satisfy p
- Shorthand for $\sigma_p(R \times S)$

$\text{User} \bowtie_{\boxed{\text{User.uid} = \text{Member.uid}}} \text{Member}$

Join example

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

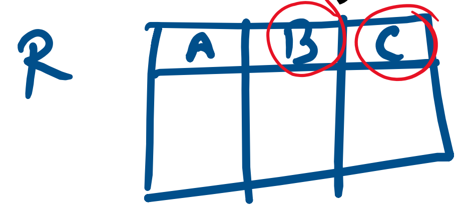
uid	gid
123	gov
857	abc
857	gov
…	…

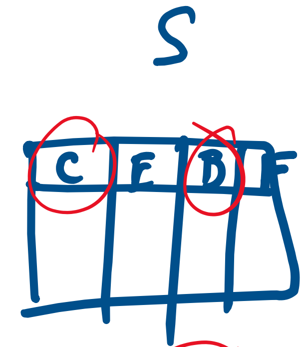
$\bowtie_{User.uid=Member.uid}$

uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
[redacted]					
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

Prefix a column reference with table name and "." to disambiguate identically named columns from different tables

Derived operator: natural join

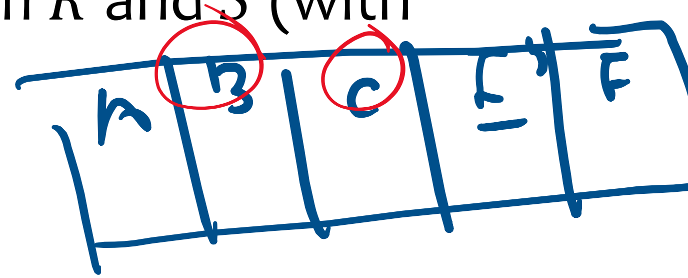




- Input: two tables R and S
- Notation: $R \bowtie S$

$R \bowtie S \quad (R.C = S.C) \wedge (R.B = S.B)$

- Purpose: relate rows from two tables, and
  - Enforce equality between identically named columns
  - Eliminate one copy of identically named columns
- Shorthand for $\pi_L\left(R \bowtie_p S\right)$, where
  - p equates each pair of columns common to R and S
  - L is the union of column names from R and S (with duplicate columns removed)



Natural join example

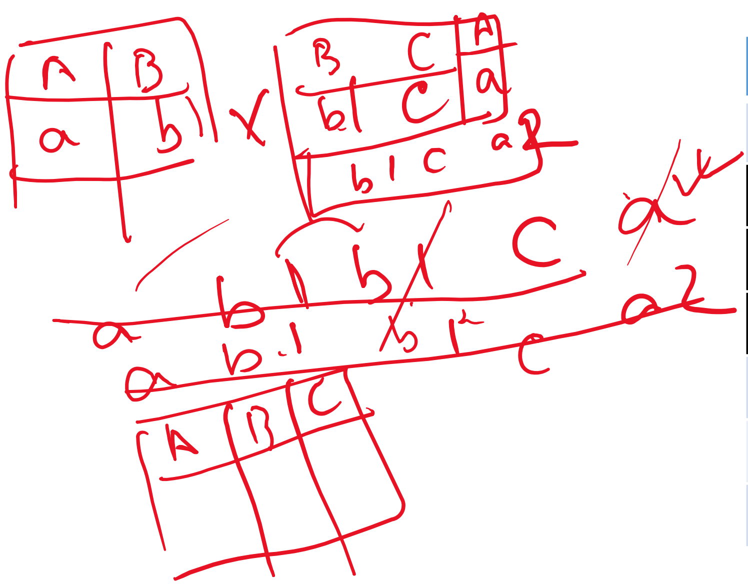
$$User \bowtie Member = \pi_{?}(User \bowtie_{?} Member)$$
$$= \pi_{uid,name,age,pop,gid}\left(User \bowtie_{\substack{User.uid= \\ Member.uid}} Member\right)$$

uid	*name*	*age*	*pop*
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	*gid*
123	gov
857	abc
857	gov
…	…

$\bowtie$

uid	*name*	*age*	*pop*	[redacted]	*gid*
123	Milhouse	10	0.2	[redacted]	gov
[redacted]					
857	Lisa	8	0.7	[redacted]	abc
857	Lisa	8	0.7	[redacted]	gov
…	…	…	…	[redacted]	…



Union

- Input: two tables R and S
- Notation: $R \cup S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R and all rows in S (with duplicate rows removed)

R and S should be union-compatible

$\cap$ $-$

Difference

- Input: two tables R and S
- Notation: $R - S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R that are not in S

$\{1, 2, 3\}$
$- \{2, 5, 6, 7\}$
$= \{1, 3\}$

R

A	B	C
a	b	c
a	b1	c
a1	b2	c

−

S

A	B	C
a1	b3	c
a1	b2	c

A	B	C
a	b	c
a	b1	c

Derived operator: intersection

- Input: two tables R and S
- Notation: $R \cap S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows that are in both R and S
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

$R \cap S$

Renaming

- Input: a table R
- Notation: $\rho_S\, R$, $\rho_{(A_1,A_2,\ldots)} R$, or $\rho_{S(A_1,A_2,\ldots)} R$
- Purpose: "rename" a table and/or its columns
- Output: a table with the same rows as R, but called differently
- Used to
  - Avoid confusion caused by identical column names
  - Create identical column names for natural joins
- As with all other relational operators, it doesn't modify the database
  - Think of the renamed table as a copy of the original

Member (uid, gid) $\rightarrow$

Renaming example

- IDs of users who belong to at least two groups

$$Member \bowtie_{?} Member$$

$$\pi_{uid}\left(Member \bowtie_{\substack{Member.uid=Member.uid \,\wedge \\ Member.gid \neq Member.gid}} Member\right)$$

WRONG!

$\rho_{uid \rightarrow uid1}$ Member

$$\pi_{uid_1}\left(\begin{array}{c}\rho_{(uid_1,gid_1)} Member \\ \bowtie_{uid_1=uid_2 \,\wedge\, gid_1 \neq gid_2} \\ \rho_{(uid_2,gid_2)} Member\end{array}\right)$$

Expression tree notation

Also called logical
Plan tree

$$\pi_{uid_1}$$

$$\bowtie_{uid_1=uid_2 \land gid_1 \neq gid_2}$$

$$\rho_{(uid_1, gid_1)} \qquad \rho_{(uid_2, gid_2)}$$

$$Member \qquad Member$$

Summary of core operators

- Selection: $\sigma_p R$
- Projection: $\pi_L R$
- Cross product: $R \times S$
- Union: $R \cup S$
- Difference: $R - S$
- Renaming: $\rho_{S(A_1, A_2, \dots)} R$
    - Does not really add "processing" power

Summary of derived operators

- Join: $R \bowtie_p S$
- Natural join: $R \bowtie S$
- Intersection: $R \cap S$

- Many more
  - Semijoin, anti-semijoin, quotient, …

User (uid, name, age, pop)
Group (gid, name)
Member (uid, gid)

An exercise

- Names of users in Lisa's groups

Writing a query bottom-up:

Π_{name}

Their names

$\bowtie_{uid}$ User

Users in Lisa's groups

$\bowtie_{gid=gid}$ Member

Lisa's groups π_{gid}

$\bowtie$

Who's Lisa? $\sigma_{name="Lisa"}$ User

Member

uid	n	a	p
7	Paul		
5	L		
9	J.		

M

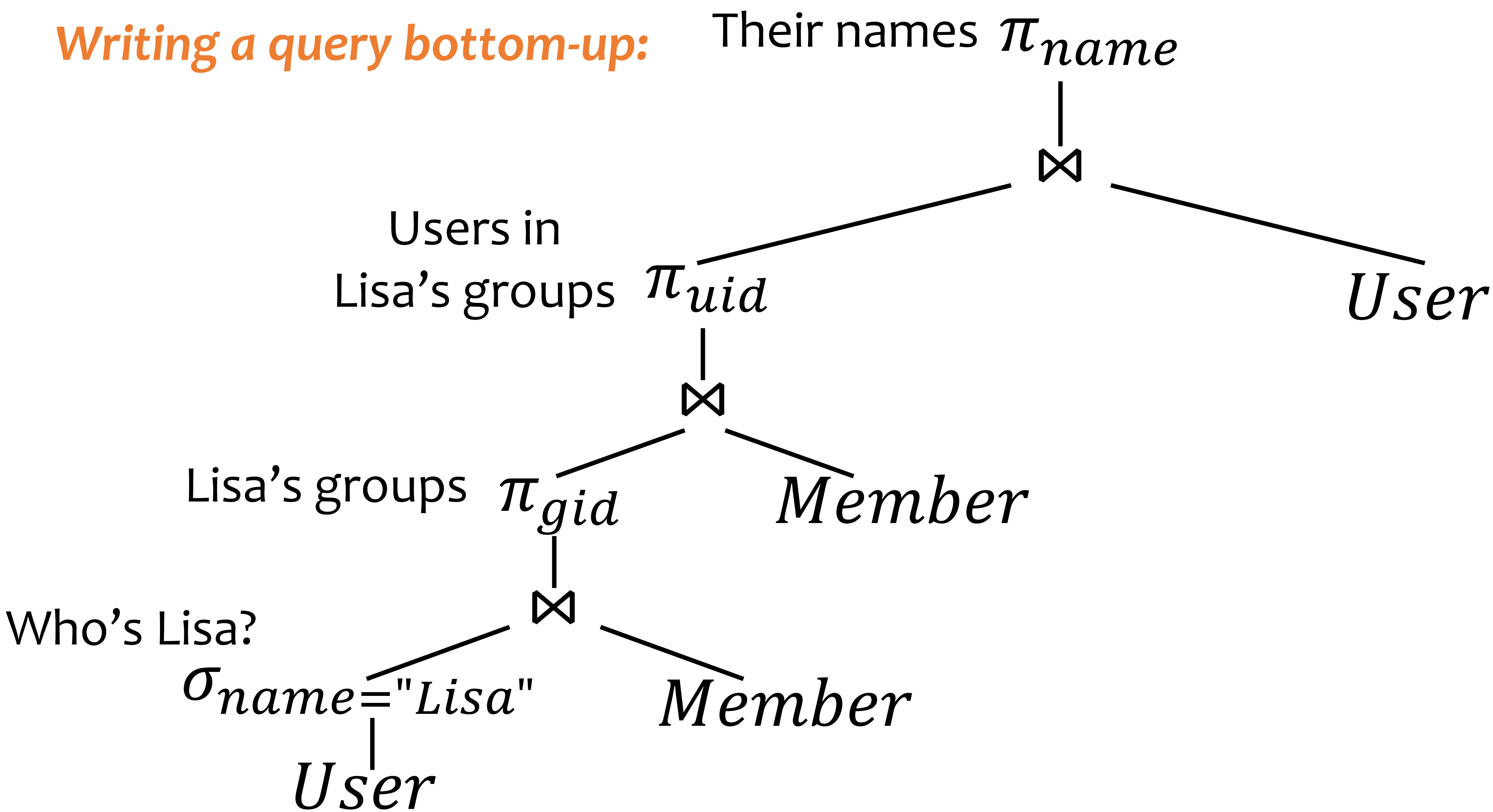
uid	gid
5	abc
5	gov
7	abc
9	gov
10	gov

An exercise

User (uid, name, age, pop)
Group (gid, name)
Member (uid, gid)

- Names of users in Lisa's groups

Writing a query bottom-up:



User (uid, name, age, pop)
Group (gid, name)
Member (uid, gid)

Another exercise

"NOT"

- IDs of groups that Lisa doesn't belong to

Writing a query top-down:

All groups − Lisa belongs to

$$\pi_{gid}\,\text{Group} \;-\; \pi_{gid}\left(\sigma_{n=Lisa}\,\text{User} \bowtie \text{Member}\right)$$

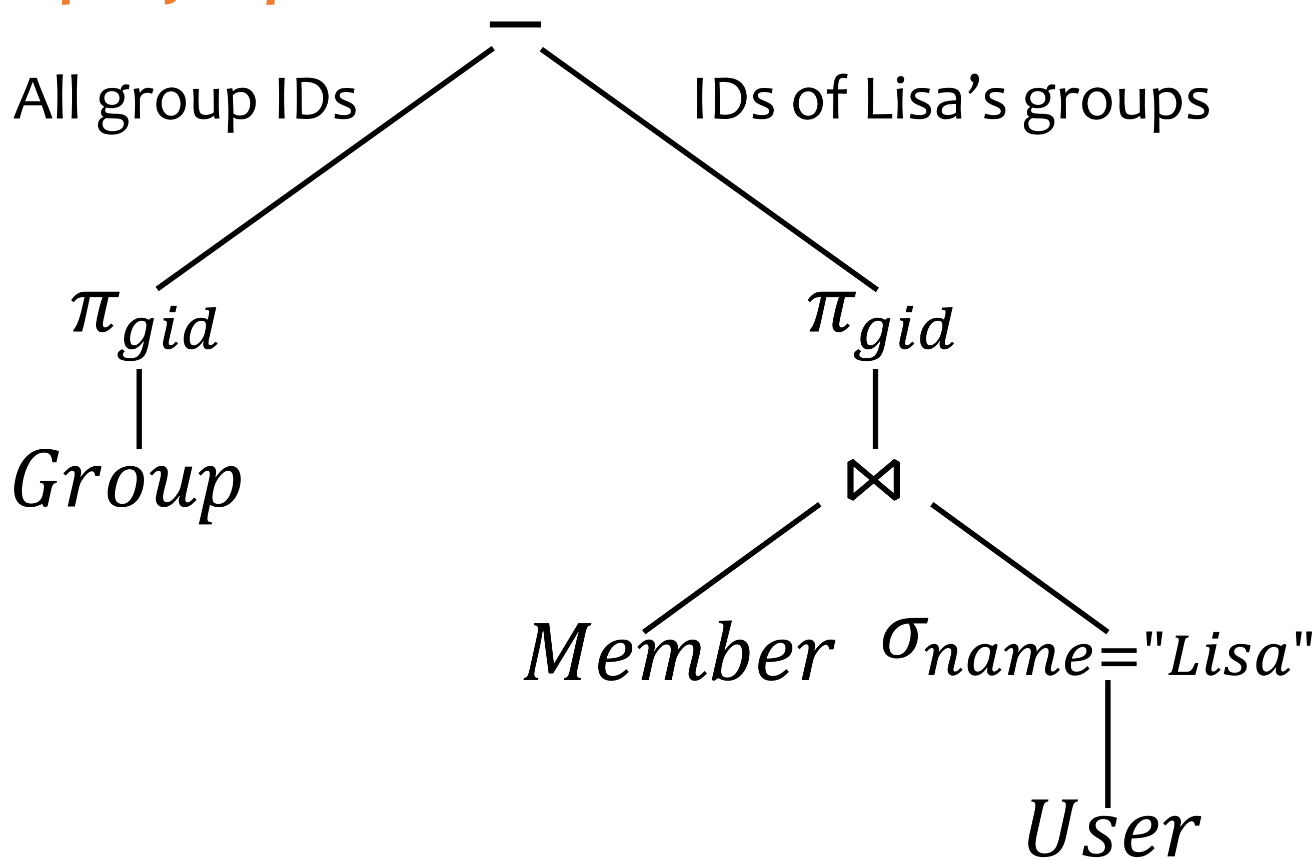
union-compatible
same schema

User (uid, name, age, pop)
Group (gid, name)
Member (uid, gid)

Another exercise

- IDs of groups that Lisa doesn't belong to

Writing a query top-down:



User (uid, name, age, pop)
Group (gid, name)
Member (uid, gid)

A trickier exercise

- Who are the most popular?
  - Who do NOT have the highest *pop* rating?
  - Whose *pop* is lower than somebody else's?

User (uid, name, age, pop)
Group (gid, name)
Member (uid, gid)

A trickier exercise

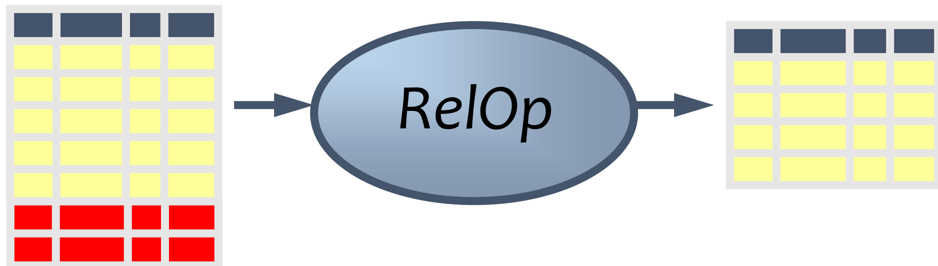
- Who are the most popular?
  - Who do NOT have the highest *pop* rating?
  - Whose *pop* is lower than somebody else's?

$$\pi_{uid}\, User - \pi_{User_1.uid}\left(\rho_{User_1} User \bowtie_{User_1.pop < User_2.pop} \rho_{User_2} User\right)$$

A deeper question:
When (and why) is "–" needed?

Monotone operators

Add more rows to the input...



What happens to the output?

- If some old output rows may need to be removed
  - Then the operator is non-monotone
- Otherwise the operator is monotone
  - That is, old output rows always remain “correct” when more rows are added to the input
- Formally, for a monotone operator op:
  $R \subseteq R'$ implies $op(R) \subseteq op(R')$ for any R, R'

Classification of relational operators

• Selection: $\sigma_p R$	Monotone
• Projection: $\pi_L R$	Monotone
• Cross product: $R \times S$	Monotone
• Join: $R \bowtie_p S$	Monotone
• Natural join: $R \bowtie S$	Monotone
• Union: $R \cup S$	Monotone
• Difference: $R - S$	Monotone w.r.t. R; non-monotone w.r.t S
• Intersection: $R \cap S$	Monotone

Why is "—" needed for "highest"?

- Composition of monotone operators produces a monotone query
  - Old output rows remain "correct" when more rows are added to the input
- Is the "highest" query monotone?
  - No!
  - Current highest *pop* is 0.9
  - Add another row with *pop* 0.91
  - Old answer is invalidated

☞ So it must use difference!

Extensions to relational algebra

- Duplicate handling (“bag algebra”)
- Grouping and aggregation
- “Extension” (or “extended projection”) to allow new column values to be computed

☞All these will come up when we talk about SQL

☞But for now we will stick to standard relational algebra without these extensions

Why is RA a good query language?

- Simple
  - A small set of core operators
  - Semantics are easy to grasp
- Declarative?
  - Yes, compared with older languages like CODASYL
  - Though operators do look somewhat “procedural”
- Complete?
  - With respect to what?

Relational calculus

- $\{u.uid \mid u \in User \land$
  $\neg(\exists u' \in User: u.pop < u'.pop)\}$, or
- $\{u.uid \mid u \in User \land$
  $(\forall u' \in User: u.pop \geq u'.pop)\}$
- Relational algebra = "safe" relational calculus
  - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
  - And vice versa
- Example of an "unsafe" relational calculus query
  - $\{u.name \mid \neg(u \in User)\}$
  - Cannot evaluate it just by looking at the database

Turing machine

- A conceptual device that can execute any computer algorithm
- Approximates what general-purpose programming languages can do
  - E.g., Python, Java, C++, …

Alan Turing (1912-1954)

☞So how does relational algebra compare with a Turing machine?

http://en.wikipedia.org/wiki/File:Alan_Turing_photo.jpg

Limits of relational algebra

- Relational algebra has no recursion
  - Example: given relation *Friend*(*uid1*, *uid2*), who can Bart reach in his social network with any number of hops?
    - Writing this query in RA is impossible!
  - So RA is not as powerful as general-purpose languages
- But why not?
  - Optimization becomes undecidable
  - ☞Simplicity is empowering
  - Besides, you can always implement it at the application level, and recursion is added to SQL nevertheless!