

Relational Model and Algebra

Introduction to Databases

CompSci 316 Spring 2019



DUKE
COMPUTER SCIENCE

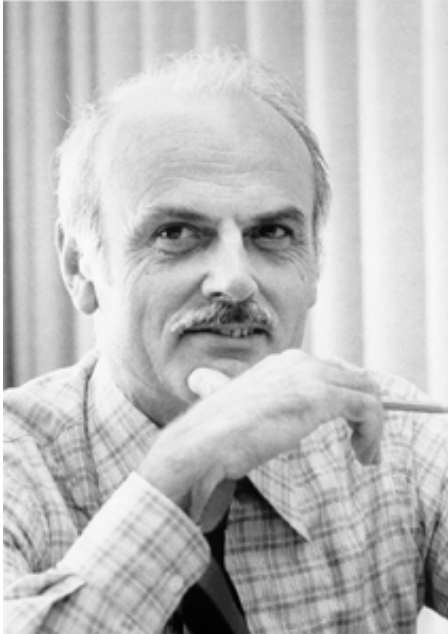
Announcements (Tue. Jan. 15)

- You should be on Piazza!
 - Otherwise, let me know after class
- HW1 to be posted today, due in about 2.5 weeks
 - Problems will be posted one by one after the material is covered in class (and announced on piazza)
 - Keep working on them
- Sign up for gradiance and gradescope
 - Tokens posted on Piazza

Announcements (Tue. Jan. 15)

- Set up VM
 - Instructions on course website
 - Google cloud coupon will be sent soon
 - There will be Help session next week
- TA/UTA office hours to be posted soon

Edgar F. Codd (1923-2003)



- Pilot in the Royal Air Force in WW2
- Inventor of the relational model and algebra while at IBM
- Turing Award, 1981

Relational data model

- A database is a collection of **relations** (or **tables**)
- Each relation has a set of **attributes** (or **columns**)
- Each attribute has a name and a **domain** (or **type**)
 - Set-valued attributes are not allowed
- Each relation contains a set of **tuples** (or **rows**)
 - Each tuple has a value for each attribute of the relation
 - Duplicate tuples are not allowed
 - Two tuples are duplicates if they agree on all attributes

☞ Simplicity is a virtue!

Example

User

<i>uid</i>	<i>name</i>	<i>age</i>	<i>pop</i>
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
...

Ordering of rows doesn't matter
(even though output is
always in some order)

Group

<i>gid</i>	<i>name</i>
abc	Book Club
gov	Student Government
dps	Dead Putting Society
...	...

Member

<i>uid</i>	<i>gid</i>
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
...	...

Schema vs. instance

- **Schema (metadata)**
 - Specifies the logical structure of data
 - Is defined at setup time
 - Rarely changes
 - **Instance**
 - Represents the data content
 - Changes rapidly, but always conforms to the schema
- 👉 Compare to **types** vs. collections of **objects of these types** in a programming language

Example

- Schema

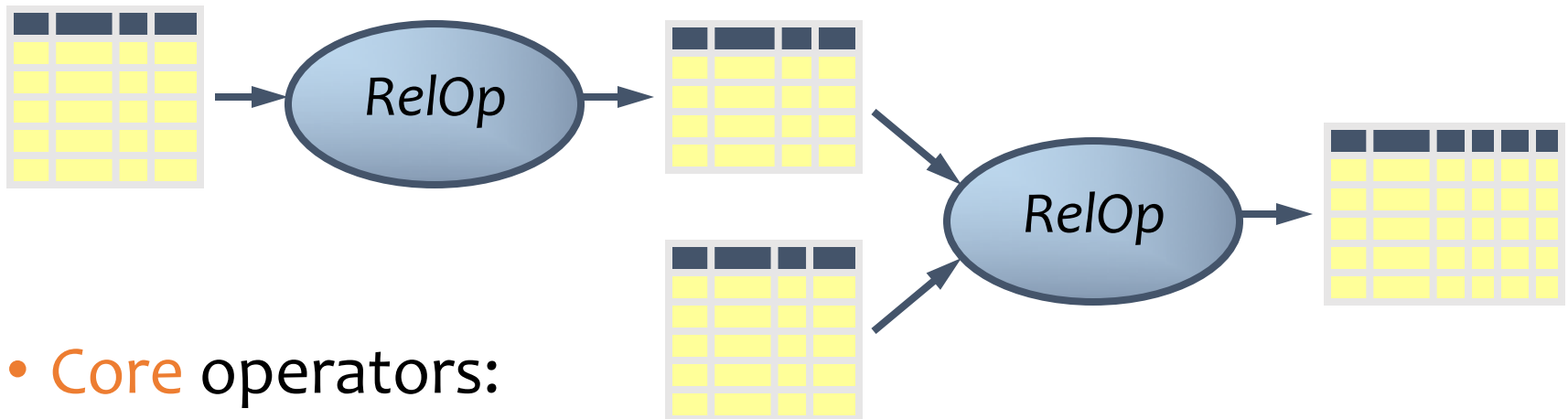
- *User* (*uid* int, *name* string, *age* int, *pop* float)
- *Group* (*gid* string, *name* string)
- *Member* (*uid* int, *gid* string)

- Instance

- *User*: {⟨142, Bart, 10, 0.9⟩, ⟨857, Milhouse, 10, 0.2⟩, ... }
- *Group*: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, ... }
- *Member*: {⟨142, dps⟩, ⟨123, gov⟩, ... }

Relational algebra

A language for querying relational data based on “operators”



- **Core** operators:

- Selection, projection, cross product, union, difference, and renaming

- Additional, **derived** operators:

- Join, natural join, intersection, etc.
- Compose operators to make complex queries

Selection

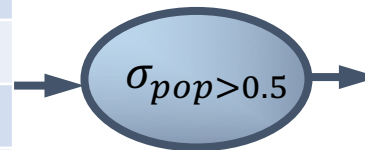
- Input: a table R
- Notation: $\sigma_p R$
 - p is called a **selection condition** (or **predicate**)
- Purpose: filter rows according to some criteria
- Output: same columns as R , but only rows of R that satisfy p

Selection example

- Users with popularity higher than 0.5

$$\sigma_{pop>0.5} User$$

<i>uid</i>	<i>name</i>	<i>age</i>	<i>pop</i>
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
...



<i>uid</i>	<i>name</i>	<i>age</i>	<i>pop</i>
142	Bart	10	0.9
857	Lisa	8	0.7
...

No actual deletion!

More on selection

- Selection condition can include any column of R , constants, comparison ($=$, \leq , etc.) and Boolean connectives (\wedge : and, \vee : or, \neg : not)
 - Example: users with popularity at least 0.9 and age under 10 or above 12

$$\sigma_{pop \geq 0.9 \wedge (age < 10 \vee age > 12)} User$$

- You must be able to evaluate the condition over **each single row** of the input table!
 - Example: the most popular user

$$\sigma_{pop \geq \text{every pop in } User} User$$

WRONG!

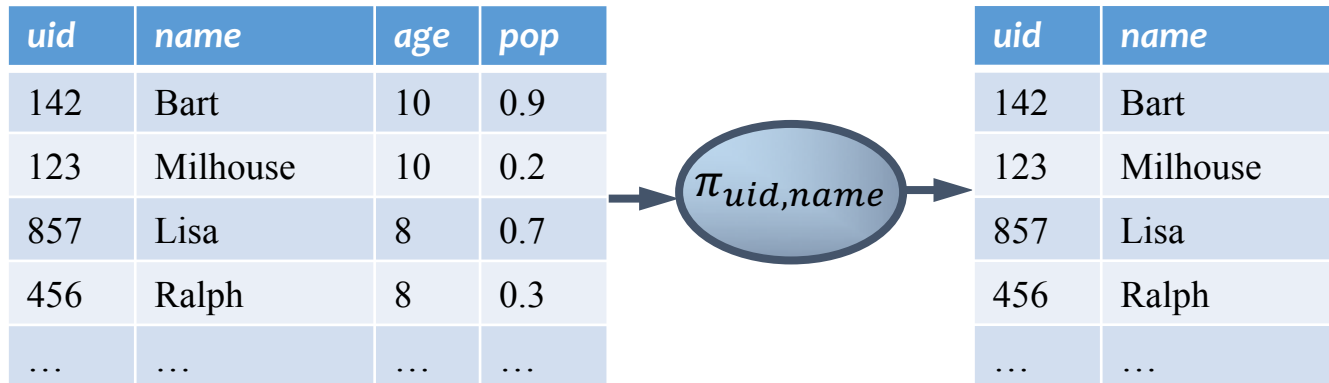
Projection

- Input: a table R
- Notation: $\pi_L R$
 - L is a list of columns in R
- Purpose: output chosen columns
- Output: same rows, but only the columns in L

Projection example

- IDs and names of all users

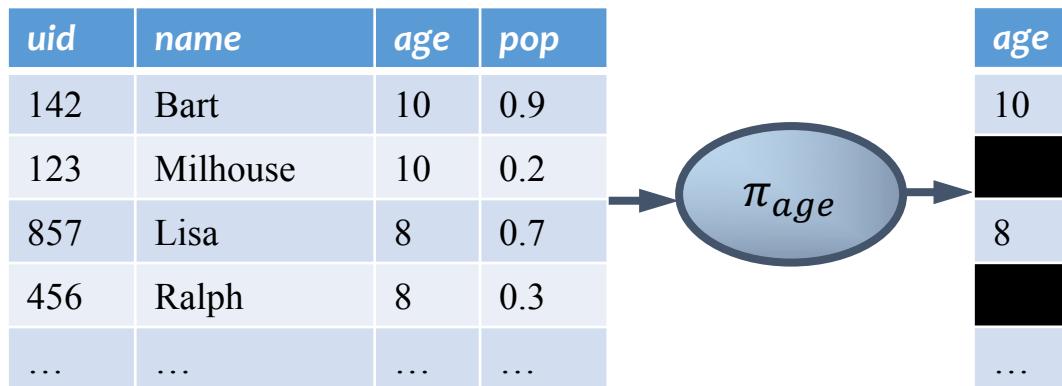
$\pi_{uid,name} User$



More on projection

- Duplicate output rows are removed (by definition)
 - Example: user ages

$\pi_{age} User$

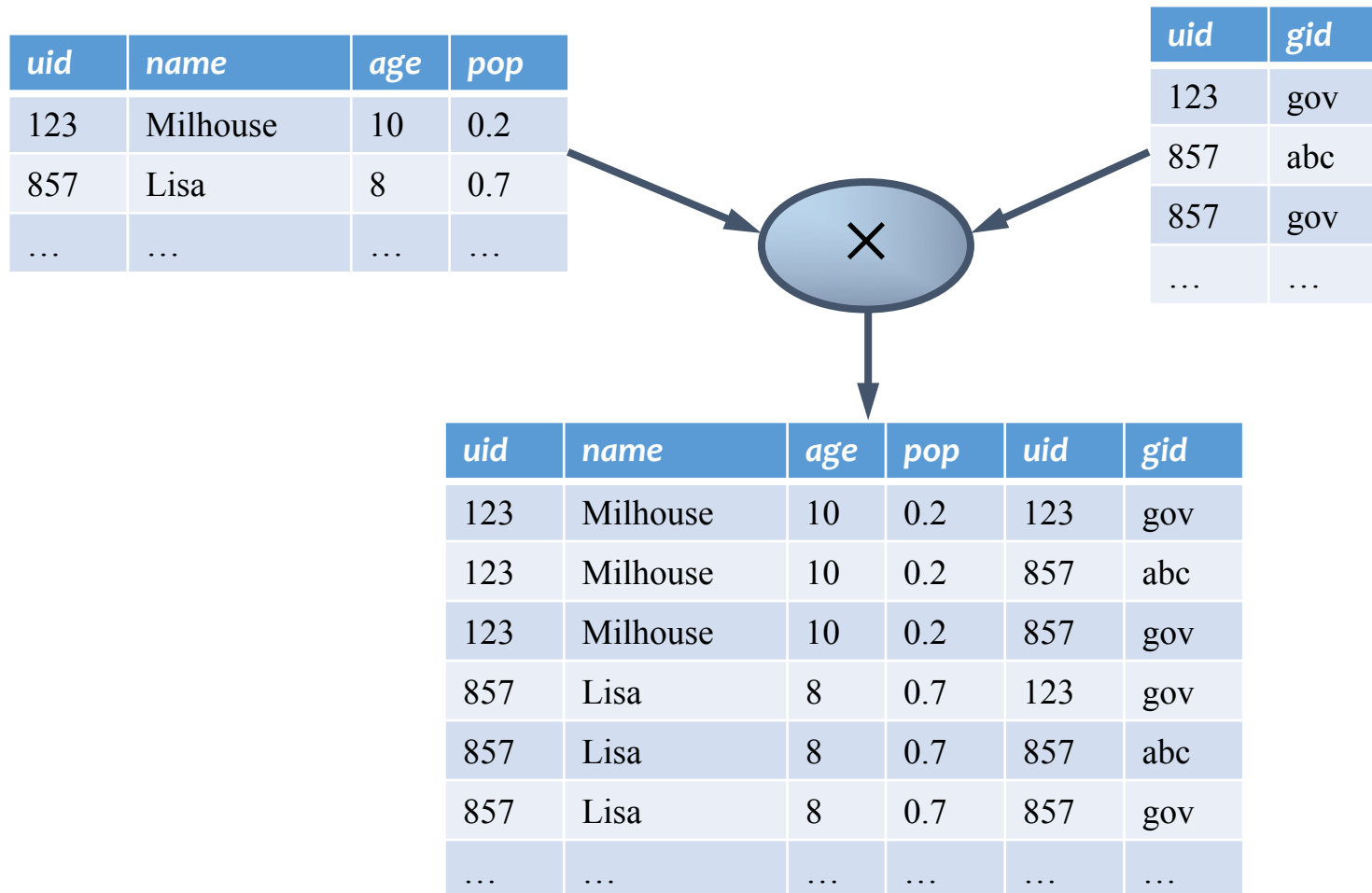


Cross product

- Input: two tables R and S
- Notation: $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row r in R and each s in S , output a row rs (concatenation of r and s)

Cross product example

User × *Member*



A note a column ordering

- Ordering of columns is unimportant as far as contents are concerned

<i>uid</i>	<i>name</i>	<i>age</i>	<i>pop</i>	<i>uid</i>	<i>gid</i>
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
...

=

<i>uid</i>	<i>gid</i>	<i>uid</i>	<i>name</i>	<i>age</i>	<i>pop</i>
123	gov	123	Milhouse	10	0.2
857	abc	123	Milhouse	10	0.2
857	gov	123	Milhouse	10	0.2
123	gov	857	Lisa	8	0.7
857	abc	857	Lisa	8	0.7
857	gov	857	Lisa	8	0.7
...

- So cross product is **commutative**, i.e., for any R and S , $R \times S = S \times R$ (up to the ordering of columns)

Derived operator: join

(A.k.a. “theta-join”)

- Input: two tables R and S
- Notation: $R \bowtie_p S$
 - p is called a **join condition** (or **predicate**)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row r in R and each row s in S , output a row rs if r and s satisfy p
- Shorthand for $\sigma_p(R \times S)$

Join example

- Info about users, plus IDs of their groups

$User \bowtie_{User.uid=Member.uid} Member$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
...

uid	gid
123	gov
857	abc
857	gov
...	...



uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
...

Prefix a column reference with table name and "." to disambiguate identically named columns from different tables

Derived operator: natural join

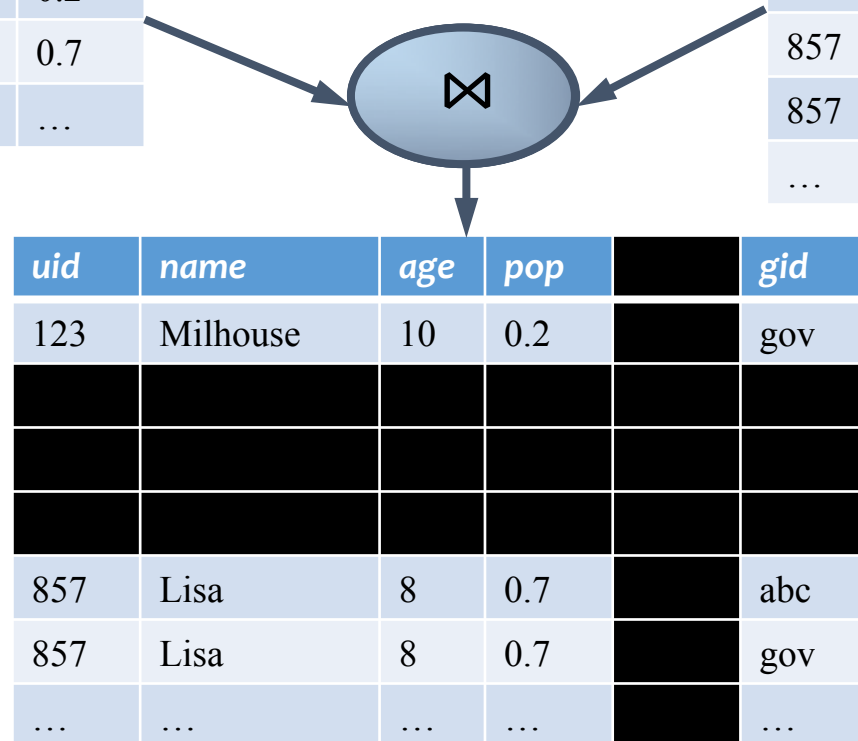
- Input: two tables R and S
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
 - Enforce equality between identically named columns
 - Eliminate one copy of identically named columns
- Shorthand for $\pi_L(R \bowtie_p S)$, where
 - p equates each pair of columns common to R and S
 - L is the union of column names from R and S (with duplicate columns removed)

Natural join example

$$\begin{aligned}
 User \bowtie Member &= \pi_{?}(User \bowtie_{?} Member) \\
 &= \pi_{uid, name, age, pop, gid} \left(User \bowtie_{\substack{User.uid = \\ Member.uid}} Member \right)
 \end{aligned}$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
...

uid	gid
123	gov
857	abc
857	gov
...	...



Union

- Input: two tables R and S
- Notation: $R \cup S$
 - R and S must have identical schema
- Output:
 - Has the same schema as R and S
 - Contains all rows in R and all rows in S (with duplicate rows removed)

Difference

- Input: two tables R and S
- Notation: $R - S$
 - R and S must have identical schema
- Output:
 - Has the same schema as R and S
 - Contains all rows in R that are not in S

Derived operator: intersection

- Input: two tables R and S
- Notation: $R \cap S$
 - R and S must have identical schema
- Output:
 - Has the same schema as R and S
 - Contains all rows that are in both R and S
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

Renaming

- Input: a table R
- Notation: $\rho_S R$, $\rho_{(A_1, A_2, \dots)} R$, or $\rho_{S(A_1, A_2, \dots)} R$
- Purpose: “rename” a table and/or its columns
- Output: a table with the same rows as R , but called differently
- Used to
 - Avoid confusion caused by identical column names
 - Create identical column names for natural joins
- As with all other relational operators, it doesn't modify the database
 - Think of the renamed table as a copy of the original

Renaming example

- IDs of users who belong to at least two groups

$Member \bowtie_? Member$

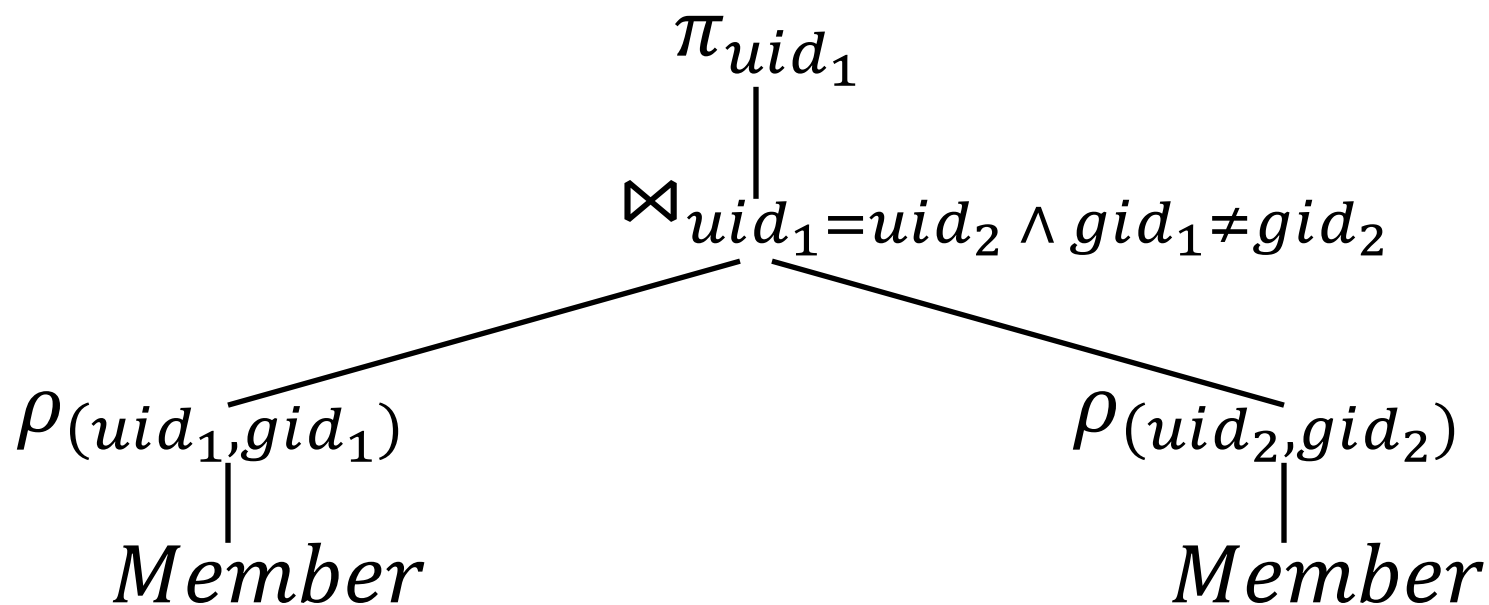
$$\pi_{uid} \left(Member \bowtie_{\substack{Member.uid=Member.uid \wedge \\ Member.gid \neq Member.gid}} Member \right)$$

WRONG!

$$\pi_{uid_1} \left(\begin{array}{c} \rho_{(uid_1, gid_1)} Member \\ \bowtie_{uid_1=uid_2 \wedge gid_1 \neq gid_2} \\ \rho_{(uid_2, gid_2)} Member \end{array} \right)$$

Expression tree notation

Also called logical
Plan tree



Summary of core operators

- Selection: $\sigma_p R$
- Projection: $\pi_L R$
- Cross product: $R \times S$
- Union: $R \cup S$
- Difference: $R - S$
- Renaming: $\rho_{S(A_1, A_2, \dots)} R$
 - Does not really add “processing” power

Summary of derived operators

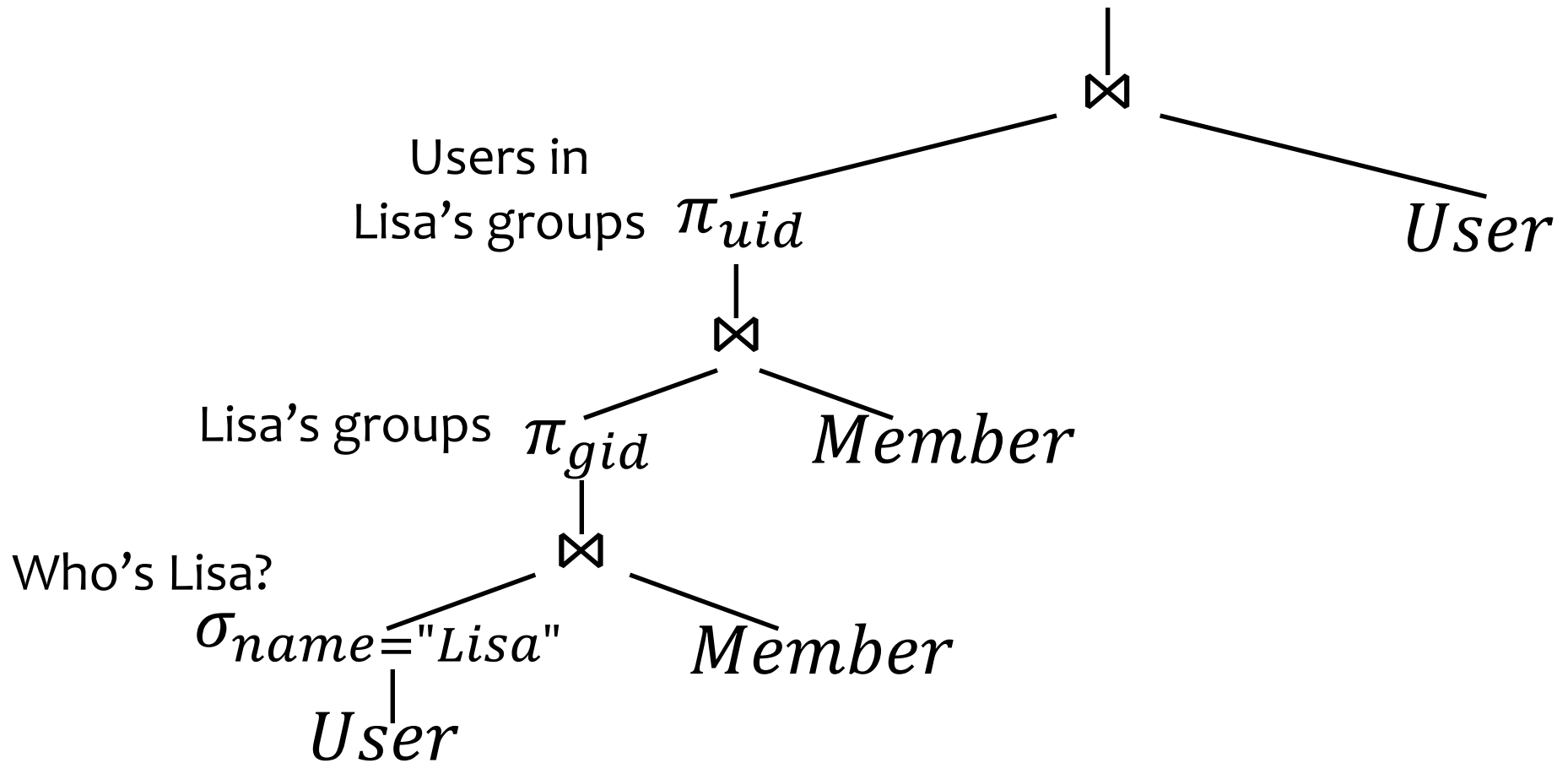
- Join: $R \bowtie_p S$
- Natural join: $R \bowtie S$
- Intersection: $R \cap S$
- Many more
 - Semijoin, anti-semijoin, quotient, ...

An exercise

User (uid, name, age, pop)
Group (gid, name)
Member (uid, gid)

- Names of users in Lisa's groups

Writing a query bottom-up: Their names π_{name}

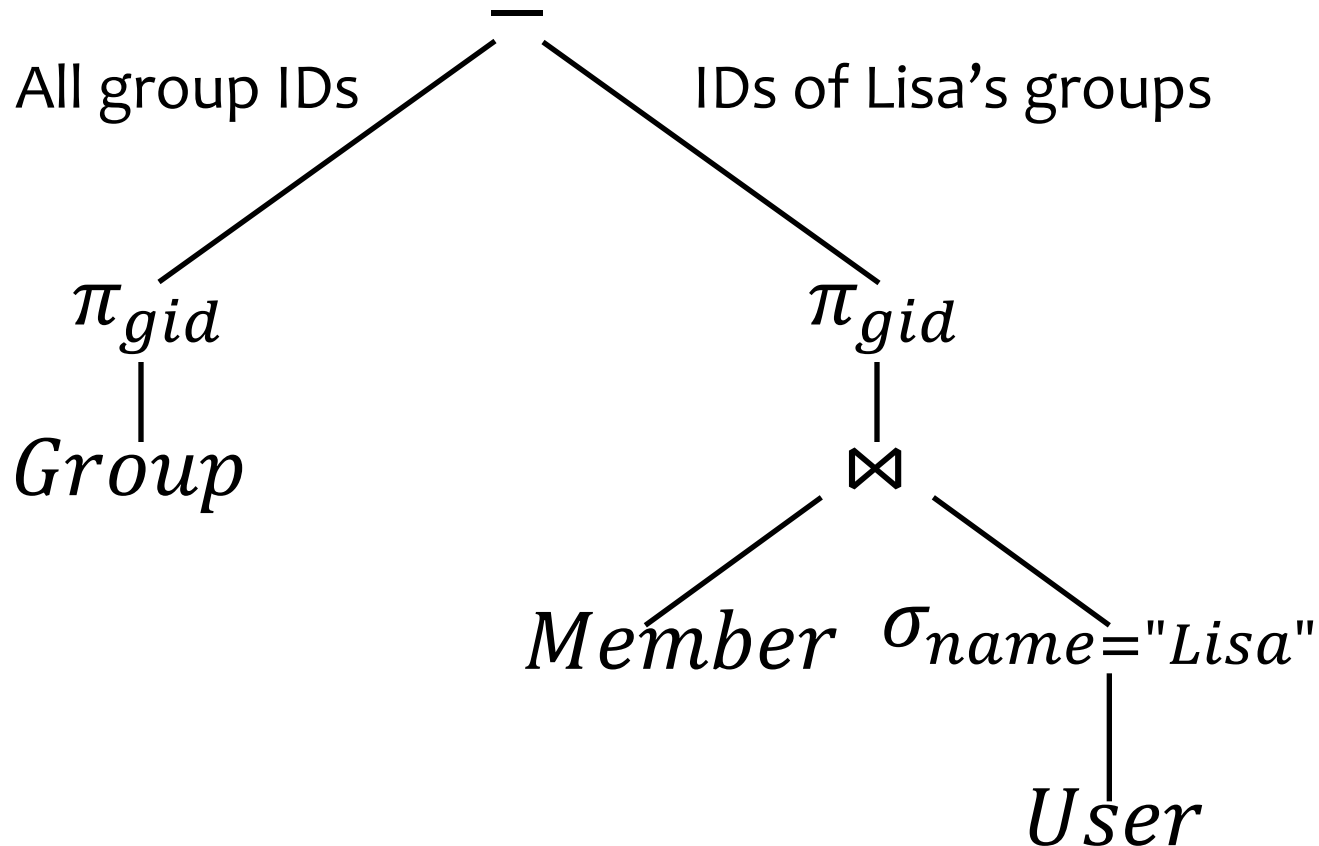


User (uid, name, age, pop)
Group (gid, name)
Member (uid, gid)

Another exercise

- IDs of groups that Lisa doesn't belong to

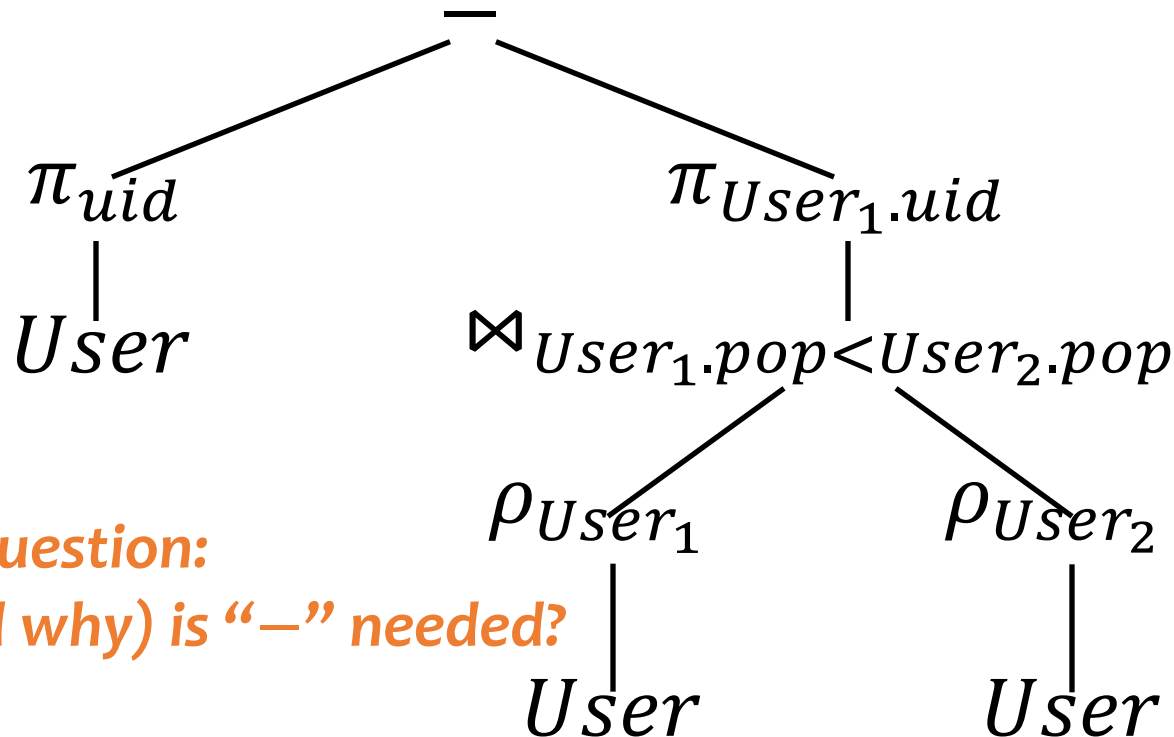
Writing a query top-down:



User (uid, name, age, pop)
 Group (gid, name)
 Member (uid, gid)

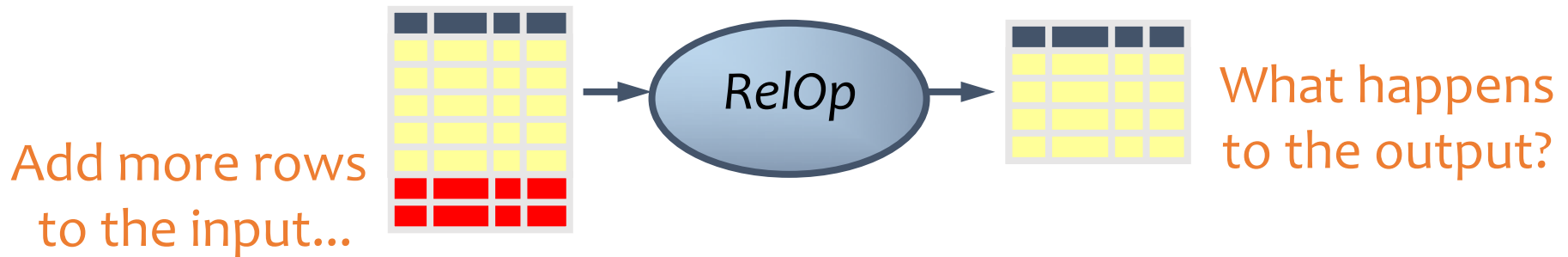
A trickier exercise

- Who are the most popular?
 - Who do NOT have the highest pop rating?
 - Whose pop is lower than somebody else's?



A deeper question:
 When (and why) is “—” needed?

Monotone operators



- If some old output rows may need to be removed
 - Then the operator is **non-monotone**
- Otherwise the operator is **monotone**
 - That is, old output rows always remain “correct” when more rows are added to the input
- Formally, for a monotone operator op :
 $R \subseteq R'$ implies $op(R) \subseteq op(R')$ for any R, R'

Classification of relational operators

- Selection: $\sigma_p R$ Monotone
- Projection: $\pi_L R$ Monotone
- Cross product: $R \times S$ Monotone
- Join: $R \bowtie_p S$ Monotone
- Natural join: $R \bowtie S$ Monotone
- Union: $R \cup S$ Monotone
- Difference: $R - S$ Monotone w.r.t. R ; non-monotone w.r.t S
- Intersection: $R \cap S$ Monotone

Why is “−” needed for “highest”?

- Composition of monotone operators produces a **monotone query**
 - Old output rows remain “correct” when more rows are added to the input
 - Is the “highest” query monotone?
 - No!
 - Current highest *pop* is 0.9
 - Add another row with *pop* 0.91
 - Old answer is invalidated
- ☞ So it must use difference!

Extensions to relational algebra

- Duplicate handling (“bag algebra”)
- Grouping and aggregation
- “Extension” (or “extended projection”) to allow new column values to be computed

☞ All these will come up when we talk about SQL

☞ But for now we will stick to standard relational algebra without these extensions

Why is RA a good query language?

- Simple
 - A small set of core operators
 - Semantics are easy to grasp
- Declarative?
 - Yes, compared with older languages like CODASYL
 - Though operators do look somewhat “procedural”
- Complete?
 - With respect to what?

Relational calculus

- $\{u.uid \mid u \in User \wedge \neg(\exists u' \in User: u.pop < u'.pop)\}$, or
- $\{u.uid \mid u \in User \wedge (\forall u' \in User: u.pop \geq u'.pop)\}$
- Relational algebra = “safe” relational calculus
 - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
 - And vice versa
- Example of an “unsafe” relational calculus query
 - $\{u.name \mid \neg(u \in User)\}$
 - Cannot evaluate it just by looking at the database

Turing machine

- A conceptual device that can execute any computer algorithm
- Approximates what **general-purpose programming languages** can do
 - E.g., Python, Java, C++, ...



Alan Turing (1912-1954)

☞ So how does relational algebra compare with a Turing machine?

Limits of relational algebra

- Relational algebra has **no recursion**
 - Example: given relation *Friend*(uid1, uid2), who can Bart reach in his social network with any number of hops?
 - Writing this query in RA is impossible!
 - So RA is not as powerful as general-purpose languages
- But why not?
 - Optimization becomes **undecidable**
 - ☞ Simplicity is empowering
 - Besides, you can always implement it at the application level, and recursion is added to SQL nevertheless!