Write and justify your answers in the space provided. ¹

1. (CLRS 6.1-1) What are the minimum and maximum number of elements in a heap of height $h$?

2. (CLRS 6.1-4) Where in a max-heap might the smallest element reside, assuming that all elements are distinct?

3. (CLRS 6.2-4) What is the effect of calling MAX-HEAPIFY($A, i$) for $i > \text{size}[A]/2$?

¹Collaboration is allowed, even encouraged, provided that the names of the collaborators are listed along with the solutions. Students must write up the solutions on their own.
4. (CLRS 6.5-3) Write pseudocode for the procedures HEAP-MINIMUM, HEAP-EXTRACT-MIN, HEAP-DECREASE-KEY and MIN-HEAP-INSERT that implement a min-priority queue with a min-heap.
5. (CLRS 6-2) Analysis of d-ary heaps

A d-ary heap is like a binary heap, but instead of 2 children, nodes have \( d \) children.

a. How would you represent a \( d \)-ary heap in an array?

b. What is the height of a \( d \)-ary heap of \( n \) elements in terms of \( n \) and \( d \)?

c. Give an efficient implementation of EXTRACT-MAX. Analyze its running time in terms of \( d \) and \( n \).

d. Give an efficient implementation of INSERT. Analyze its running time in terms of \( d \) and \( n \).

e. Give an efficient implementation of \texttt{HEAP-INCREASE-KEY}(A, i, k), which sets \( A[i] \leftarrow \max(A[i], k) \) and updates the heap structure appropriately. Analyze its running time in terms of \( d \) and \( n \).