CPS 130 Homework 1 - Solutions

1. (CLRS 1.2-2) Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size $n$, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \log n$ steps. For which values of $n$ does insertion sort beat merge sort?

**Solution:** We want to find $n$ such that $8n^2 \leq 64n \log n$:

\[
8n^2 \leq 64n \log n \Rightarrow n \leq 8 \log n
\]

\[
\Rightarrow \frac{1}{8} \leq \frac{\log n}{n},
\]

which is satisfied for $n \leq 43$.

2. (CLRS 1-1) For each function $f(n)$ and time $t$ in the following table, determine the largest size $n$ of a problem that can be solved in time $t$, assuming that the algorithms to solve the problem takes $f(n)$ microseconds.

**Solution:**

<table>
<thead>
<tr>
<th>$n$</th>
<th>1 second</th>
<th>1 minute</th>
<th>1 day</th>
<th>1 month</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^6$</td>
<td>$60 \times 10^6$</td>
<td>$864 \times 10^6$</td>
<td>$26 \times 10^{11}$</td>
<td></td>
</tr>
<tr>
<td>$10^3$</td>
<td>$7745$</td>
<td>$29 \times 10^4$</td>
<td>$16 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>$2^n$</td>
<td>20</td>
<td>26</td>
<td>36</td>
<td>41</td>
</tr>
</tbody>
</table>

3. (CLRS 2.1-2) How do you modify the $\text{INSERTION - SORT}$ procedure to sort into non-increasing instead of non-decreasing order?

**Solution:** Modify the while loop test as follows: while $i > 0$ and $A[i] < \text{key}$