1. Prove the following: (20 points)

- \( n^2 \log n + 3 \log n - 100n \) is \( O(n^4) \).
- \( n^2 \log n + 3 \log n - 100n \) is \( \Theta(n^2 \log n) \).
- \( \sum_{i=1}^{n} \frac{1}{i} \in \Theta(\log n) \). This is also known as the Harmonic Series.
- Using induction, show that \( \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} \).

2. CLRS Problem 3-3a. (30 points) Hint: Many of these formulae simplify!

3. Solve the following recurrence relations. Give tight \( \Theta \) bounds, if possible, or explain why they aren’t possible. Assume that \( T(1) = 1 \). (30 points)

- \( T(n) = nT(n-1) \)
- \( T(n) = T(n/2) + \log n \)
- \( T(n) = 2T(n/2) + n \)
- \( T(n) = 2T(2n/3) + n^2 \)
- \( T(n) = 5T(n/8) + 3n \)
- \( T(n) = 3T(n-5) + 1 \) Assume that \( T(1..4) = 1 \)

4. CLRS Problem 2-4 (20 points)