1. (10 pts) Complete or answer the following. If the resulting answer has less than 10 items, just list the items. Otherwise give a description of the language, preferably formal.

\[ L_1 = \{a, b, c\} \]
\[ L_2 = ba^* \]
\[ L_3 = a^*b^*c^* \]
\[ L_4 = \{w \in \Sigma^* | n_a(w) \text{ is even}\}, \Sigma = \{a, b, c\} \]
\[ L_5 = \{w \in \Sigma^* | n_a(w) > n_b(w)\}, \Sigma = \{a, b, c\} \]

(a) \( |2L_1| = \)
(b) \( L_1 \times L_1 = \)
(c) \( |L_3 \cap L_2| = \)
(d) \( L_3 \cap L_5 = \)
(e) \( \overline{L_4} = \)

2. (16 pts) Answer TRUE or FALSE to each of the statements below.

(a) Using the pumping lemma to prove a language is not regular is a type of proof by induction. (TRUE or FALSE?)
(b) \( \{a\} \in \{\{a\}\} \). (TRUE or FALSE?)
(c) If \( M_1 \) is a DFA, then there exists an NPDA \( M_2 \) such that \( L(M_1) = L(M_2) \). (TRUE or FALSE?)
(d) If \( M \) is a CFG, then there exists a regular expression \( E \) such that \( L(M) = L(E) \). (TRUE or FALSE?)
(e) \( L = \{a^n b^n c^m | n > 0, 0 < m < 100\} \). \( L \) is regular. (TRUE or FALSE?)
(f) \( L = \{a^{2n} b^p c^{3m} | n > 0, m > 0, p > 0\} \). \( L \) is regular. (TRUE or FALSE?)
(g) \( L = \{w \in \Sigma^* | n_a(w) \text{ is even and } n_b(w) > n_a(w)\} \Sigma = \{a, b, c\} \). \( L \) is regular. (TRUE or FALSE?)
(h) \( L = \{w \in \Sigma^* | n_a(w) > 10, n_b(w) < 1000, n_a(w) = n_b(w)\} \Sigma = \{a, b\} \). \( L \) is regular. (TRUE or FALSE?)
3. (3 pts) In the proof to convert an NPDA to a CFG, the NPDA was first converted to an NPDA that 1) accepts if the stack was empty and 2) each move increases or decreases the stack content by a single symbol. Explain why this was done for the proof.

4. (3 pts) Consider the following CFG. Show that it is ambiguous.

\[ S \rightarrow aSb \mid abS \mid bS \mid aS \mid a \mid b \]

5. (8 pts) Draw a DFA for the following language. Do not show trap states. (show the transition diagram, indicate the start state by a short arrow, and final states by double circles.)

\[ L = \{ w \in \Sigma^* \mid w \text{ has at most 2 } b\text{'s and } w \text{ has the substring } aa \}, \Sigma = \{ a, b \}. \]

For example, \( ababa, aab \) and \( aaaa \) are in \( L \).

6. (6 pts) Write a NFA or DFA for the following regular grammar.

\[
\begin{align*}
S &\rightarrow aS \mid bB \\
B &\rightarrow bB \mid b
\end{align*}
\]

7. (6 pts) Write a context-free grammar for the following language.

\[ L = \{ a^n b^m c^m d^n \mid n > 0, m > 3 \} \]

8. (3 pts) In the proof to convert a DFA to a minimal state DFA, formally explain the meaning that “two states are distinguishable”.

9. (3 pts)

Consider the following DFA.

Show states \( q_1 \) and \( q_4 \) are distinguishable with an appropriate string. Explain.

10. (8 pts) Consider the proof of the following. For every DFA \( M \), there exists a regular expression \( E \) such that \( L(M) = L(E) \).

(a) What format must the DFA be in before any states can be removed?

(b) During the transformation, explain how a transition on the DFA can be different than a transition on a regular DFA.

(c) Explain which states can be removed and what the final format of the DFA is (when nothing else is removed).

(d) Simplify the regular expression below to as simple as possible.

\[ b^* (\emptyset + a) + a\lambda + \emptyset^* b \]
11. (10 pts) Consider \( L = \{ b^{2n} a^{3n} \mid n > 0 \} \). Draw the transition diagram for a nondeterministic pushdown automaton \( M \) that accepts \( L \) by final state. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are \( a, b; cd \) where \( a \) is the symbol on the tape, \( b \) is the symbol on top of the stack that is popped, and \( cd \) are pushed onto the stack (with \( c \) on top of \( d \)). \( Z \) is on top of the stack when \( M \) starts. ).

(a) First list 3 strings in \( L \).

(b) Now draw the transition diagram.

12. (6 pts) **Pumping Lemma:** Let \( L \) be an infinite regular language. \( \exists \) a constant \( m > 0 \) such that any \( w \in L \) with \( |w| \geq m \) can be decomposed into three parts as \( w = xyz \) with

\[
|xy| \leq m \\
|y| \geq 1 \\
xy^i z \in L \quad \text{for all } i \geq 0
\]

**Use the Pumping Lemma to prove** the language \( L \) below is not regular. 

\( L = \{ b^n a^p c^{np} \mid n > 0, p > 0 \} \), \( \Sigma = \{ a, b, c \} \). For example, \( baacc \) and \( bbaaaccccccc \in L \). 

**Proof:** (SHOW ALL STEPS! Some have been started for you.) 
Assume ___________________________
13. (8 pts) Consider the following property, Replace First a With aa And First b with bb (RFaFb). If L is a regular language, then RFaFb(L) = strings from L that have at least one a and one b, and the first a has been replaced with aa and the first b has been replaced with bb.

For example, if abba is in L, then aabbba (first a is replaced with aa and first b is replaced with bb) is in RFaFb(L). If bbb is in L, then no strings are added to RFaFb(L) (there are no a’s to replace).

Show that RFaFb(L) is a regular language.