NOTE: $n_a(w)$ means the number of a’s in the string $w$.

1. (12 pts) Complete or answer the following. If the resulting answer has less than 10 items, just list the items. Otherwise give a formal description of the language.

$L_1 = \{a, b\} \quad L_2 = \{a, c\} \quad L_3 = a^* b \quad L_4 = b^* ab^* \quad L_5 = \{w \in \Sigma^* \mid n_b(w) > n_a(w)\}, \Sigma = \{a, b, c\} \quad L_6 = \{w \in \Sigma^* \mid w \text{ has at most } 2 \text{ } b's\}, \Sigma = \{a, b, c\}$

(a) $L_1 \times L_2 =$
(b) $2^{L_2} =$
(c) $|L_3 \cap L_5| =$
(d) $L_4 \cap L_6 =$
(e) $\overline{L_6} =$
(f) $L_2^3 =$

2. (18 pts) Answer TRUE or FALSE to each of the statements below.

(a) $\emptyset \in \{a, b\}$? (TRUE or FALSE?)

(b) If a CFG $G$ is ambiguous, then there exists an NPDA $M$ such that $L(G) = L(M)$. (TRUE or FALSE?)

(c) If $M$ is an NFA, then there exists a regular grammar $G$ with at most 4 rules such that $L(M) = L(G)$. (TRUE or FALSE?)

(d) If $G$ is a CFG with at most two rules, then there exists a regular expression $R$ such that $L(M) = L(R)$. (TRUE or FALSE?)

(e) $L = \{w \in \Sigma^* \mid n_a(w) > n_b(w) \ast 2\}, \Sigma = \{a, b, c\}$. $L$ is regular. (TRUE or FALSE?)

(f) $L = \{a^n b^{2n} \mid 0 < n < 1000\}, \Sigma = \{a, b\}$. $L$ is regular. (TRUE or FALSE?)

(g) $L = \{w \in \Sigma^* \mid n_a(w) \text{ is even}, n_b(w) \text{ is odd}, n_c(w) \text{ is even} \}, \Sigma = \{a, b, c\}$. $L$ is regular. (TRUE or FALSE?)

(h) $L = \{a^n b^p c^m \mid m + p + n > 100, n > 0, m > 0, p > 0\}$. $L$ is regular. (TRUE or FALSE?)
(i) \( L = \{a^n b^{2m} c^p \mid n > 0, m > 0, p \geq 0\} \). L is regular. (TRUE or FALSE?)

3. (4 pts) Explain what each of the seven parts of the 7-tuple for an NPDA M represent, 
   \( M = (Q, \Sigma, \Gamma, \delta, q_0, z, F) \).

4. (3 pts) Consider the following theorem.
   **Theorem** Given NPDA M that accepts by final state, \( \exists \) NPDA M’ that accepts by empty stack s.t. \( L(M) = L(M') \).
   The proof starts with an NPDA M that accepts by final state such that \( M = (Q, \Sigma, \Gamma, \delta, q_0, z, F) \). Explain how to construct an equivalent NPDA that accepts by empty stack.

5. (8 pts) Draw a DFA for the following language. Do not show trap states. (show the transition diagram, indicate the start state by a short arrow, and final states by double circles.)
   \( L = \{w \in \Sigma^* \mid n_a(w) \text{ is even and } w \text{ has the substring } aba\}, \Sigma = \{a, b\}. \)
   For example, \( abaaba, aaababb \) and \( abaaa \) are in \( L \).

6. (6 pts) Write an equivalent regular grammar for the following DFA.

```
q0
  ^-- a
    |      b
  v
q1
  ^-- w
    |      a
  v
q2
```

7. (6 pts) Write a context-free grammar for the following language.
   \[ L = \{a^n b^m c^p \mid n + p > m, m > 0, n > 0, p > 0\} \]

8. (6 pts) Consider the following DFA.
   a) Show states \( q_0 \) and \( q_3 \) are distinguishable with an appropriate string. Explain.
   b) Give the states in the minimal state DFA (you do not need to show the arcs). Each state should indicate which states it represents from the original DFA. For example you could list one state as 0,1,2 if states 0, 1 and 2 in the original DFA can be combined to form a state in the minimal state DFA.
9. (10 pts) Consider \( L = \{a^mb^p \mid p > 2m, m > 0\} \). Draw the transition diagram for a nondeterministic pushdown automaton \( M \) that accepts \( L \) by final state. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are \( a, b; cd \) where \( a \) is the symbol on the tape, \( b \) is the symbol on top of the stack that is popped, and \( cd \) are pushed onto the stack (with \( c \) on top of \( d \)). \( Z \) is on top of the stack when \( M \) starts. ).

(a) First list 3 strings in \( L \).

(b) Now draw the transition diagram.

10. (6 pts) **Pumping Lemma:** Let \( L \) be an infinite regular language. \( \exists \) a constant \( m > 0 \) such that any \( w \in L \) with \( |w| \geq m \) can be decomposed into three parts as \( w = xyz \) with

\[
\begin{align*}
|xy| & \leq m \\
|y| & \geq 1 \\
xy^iz & \in L \quad \text{for all } i \geq 0
\end{align*}
\]

**Use the Pumping Lemma to prove** the language \( L \) below is not regular.

\( L = \{w \in \Sigma^* \mid n_a(w) + n_b(w) > n_c(w)\} \) \( \Sigma = \{a, b, c\} \).

\( \Sigma = \{a, b, c\} \). For example, \( accbb \) and \( cab \in L \).

**Proof:** (SHOW ALL STEPS! Some have been started for you.)
Assume   

Choose \( w = \)  

11. (4 pts) Consider the following incorrect proof by contradiction to prove a language \( L \) is not regular.

**Example** \( L = \{ w \in \Sigma^* \mid n_a(w) > n_b(w) > n_c(w) \} \)  \( \Sigma = \{ a, b, c \} \).

- **Proof:** (proof by contradiction)
  
  Assume \( L \) is regular.
  
  Let \( L_2 = \{ a^{n+1}b^n c^* \mid n > 0 \} \)
  
  Then \( L_3 = L \cap L_2 = \{ a^{n+1}b^n c^p \mid n > p, p > 0 \} \) is regular.
  
  Let \( L_4 = \{ a^n b^{n+1} c^* \mid n > 0 \} \)
  
  Then \( L_5 = L_3 \cap L_4 = \{ a^{n+2} b^{n+1} c^n \mid n > 0 \} \) is regular.
  
  Let \( L_6 = a^n b^n \)
  
  Then \( L_7 = L_5 \cap L_6 = \{ a^{n+1} b^n \mid n > 0 \} \) is regular.
  
  Then \( L_8 = L_7 \circ \{b\} = \{ a^n b^n \mid n > 0 \} \) is regular.
  
  Contradiction, since \( L_8 \) is already proven not regular. Thus, \( L \) is not regular.

Identify the first mistake in the proof.

12. (8 pts) Consider the following property, Replace_Every_Other_Symbol_With_c (REOSWc). If \( L \) is a regular language, then \( \text{REOSWc}(L) \) = strings from \( L \) that have every other symbol starting with the first symbol replaced with a \( c \).

For example, if \( caabb \) is in \( L \), then \( cacbc \) is in \( \text{REOSWc}(L) \). If \( a \) is in \( L \), then \( c \) is in \( \text{REOSWc}(L) \).

Prove that \( \text{REOSWc}(L) \) is a regular language.