1. (8 pts) Complete or answer the following.

\[ A = \{0, 4, 8, 12\} \quad B = \{3, 12, 15\} \quad C = \{1, 2\} \]

(a) \( |A \times B| = \)
(b) \( B \times C = \)
(c) \( A \cap B = \)
(d) \( 3 \in B \times C \) (TRUE or FALSE?)

2. (10 pts) Complete the following.

\[ L_1 = \{(ab)^n a^n \mid n > 0\} \]
\[ L_2 = \{b^n \mid n > 0\} \]
\[ L_3 = \{a^n \mid n > 0\} \]

(a) \( L_1 \circ L_2 = \)
(b) \( L_1 \cap L_2 = \)
(c) \( L_1 \cap L_3 = \)
(d) \( L_2 \times L_3 = \)
(e) \( L_1 \cap T_2 = \)

3. (16 pts) Answer TRUE or FALSE to each of the statements below.

(a) Given any DFA \( M_1 \), \( M_1 \) is converted into an equivalent regular expression \( r \), then \( r \) is converted into an equivalent NFA \( M_2 \), then \( M_2 \) is converted into an equivalent DFA \( M_3 \). \( M_1 \) and \( M_3 \) will have the same number of states. (TRUE or FALSE?)
(b) Consider determining if a string is in the language of a given DFA and in the language of a given regular grammar. One step in the DFA is traversing one arc and one step in the regular grammar is applying one rule. Both the DFA and the regular grammar will take the same number of steps in determining if a given string is in their language. (TRUE or FALSE?)
(c) A sentential form is a list of all the nodes in a partial derivation tree with root \( S \), listed in breadth-first order. (TRUE or FALSE?)
(d) Given any DFA \( M \), there exists an NFA \( M' \) with only 1 final state such that \( L(M) = L(M') \). (TRUE or FALSE?)
(e) Given any regular expression r, there exists a CFG G such that L(r)=L(G). (TRUE or FALSE?)
(f) \( L = \{ a^n b^m c^p \mid n > 2 \ast p, m > 0, p > 0 \} \). L is regular. (TRUE or FALSE?)
(g) \( L = \{ a^n b^m c^p \mid n \text{ is even}, m \text{ is odd}, p \text{ is odd} \} \). L is regular. (TRUE or FALSE?)
(h) \( L = \{ w w w \mid w \in \Sigma^* \} \). \( \Sigma = \{ a \} \). L is regular. (TRUE or FALSE?)

4. (8 pts) Draw a DFA for the following language. Do not show trap states. (show the transition diagram, indicate the start state by a short arrow, and final states by double circles.)

\[ L = \{ w \in \Sigma^* \mid w \text{ has an even number of } a \text{'s and at most 3 } b \text{'s} \} \], \( \Sigma = \{ a, b \} \).

5. (8 pts) Convert the following DFA to a minimum state DFA. \( \Sigma = \{ a, b \} \). Show the tree distinguishing the states and briefly explain at each level the reason for distinguishing the states. Show the resulting minimal dfa with states labeled with names from their original states (for example, combined states 3 and 4 would be called state 34). There is additional space on the next page.

6. (6 points) Write a context-free grammar for the following language.
\[ L = \{ a^n b^n c(ab)^m \mid n > 0, m > 0 \} \]

**Pumping Lemma:** Let \( L \) be an infinite regular language. \( \exists \) a constant \( m > 0 \) such that any \( w \in L \) with \( |w| \geq m \) can be decomposed into three parts as \( w = xyz \) with
\[
\begin{align*}
|xy| &\leq m \\
|y| &\geq 1 \\
x y^i z &\in L \quad \text{for all } i \geq 0
\end{align*}
\]

7. (6 pts) Use the Pumping Lemma to prove
\( \Sigma = \{ a, b, c \} \), \( L = \{ a^n b^p c^n \mid p < n, p > 0 \} \) is not regular.

**Proof:** (SHOW ALL STEPS! Some have been started for you.)
Assume
Choose \( w = \)

8. (5 points) Show the following grammar is ambiguous.
\[
S \rightarrow ASb | AS | a \\
A \rightarrow aaA | \lambda
\]

9. (5 pts) Give a regular expression for the following language.
\( L = \{ w \in \Sigma^* \mid w \text{ has an even number of } a\text{'s and at most 1 } b \} \), \( \Sigma = \{ a, b \} \)

10. (8 pts) Consider the following property, ReplaceFirstaaWitha (RFaaWa). If \( L \) is a regular language, then

\[
\text{RFaaWa}(L) = \{ w = uav \mid uaav \in L, u \in \Sigma^*, u \text{ does not have the substring } aa \text{ and } v \in \Sigma^* \}
\]

with \( \Sigma = \{ a, b \} \). In other words, \( \text{RFaaWa}(L) \) accepts a word from \( L \) with the first \( aa \) replaced by \( a \). For example, if \( aababa \in L \), then \( ababa \in \text{RFaaWa}(L) \). If \( babab \in L \), then \( babab \in \text{RFaaWa}(L) \). If \( aabab \in L \), then \( aaab \in \text{RFaaWa}(L) \).

**Prove** that the regular languages are closed under the \( \text{RFaaWa}(L) \) property. (Show all steps!)