NOTE: \( n_a(w) \) means the number of a’s in the string \( w \).

1. (14 pts) Complete or answer the following. If the resulting answer has less than 10 items, just list the items. Otherwise give a formal description of the language.

\[
L_1 = \{a\}, \Sigma = \{a, b\}
L_2 = \{a, b, c\}, \Sigma = \{a, b, c\}
L_3 = \{a^nb^n \mid n > 0\}, \Sigma = \{a, b\}
L_4 = \{w \in \Sigma^* \mid n_a(w) = 2 \cdot p, p \geq 0\}, \Sigma = \{a, b\}
\]

(a) \( L_1 \times L_3 = \)
(b) \( 2^{L_1} = \)
(c) \( L_1 \cap L_3 = \)
(d) \( L_3 \cap L_4 = \)
(e) \( |L_2| = \)
(f) \( \overline{L_1} = \)
(g) \( L_1 \circ L_2 = \)

2. (20 pts) Answer TRUE or FALSE to each of the statements below.

(a) \( a \in \{\emptyset, \{a\}, \{a, b\}\}? \) (TRUE or FALSE?)
(b) If \( L \) is a right-linear grammar \( G \), then \( L \) is a CFG? (TRUE or FALSE?)
(c) If \( M \) is a DFA with more than one final state, then there exists a DFA \( M' \) with only one final state such that \( L(M) = L(M') \). (TRUE or FALSE?)
(d) If \( G \) is a CFG with only one production, then there exists a DFA \( M \) such that \( L(G) = L(M) \). (TRUE or FALSE?)
(e) If \( G \) is a CFG and there are two different derivations of a string \( w \in L(G) \), then \( G \) is an ambiguous grammar. (TRUE or FALSE?)
(f) If \( L_1 \) and \( L_2 \) are regular, then there exists a regular expression \( r \) such that \( L(r) = L_1 \cap L_2 \). (TRUE or FALSE?)
(g) \( L = \{w \in \Sigma^* \mid n_a(w) > 1000, n_b(w) \text{ is even}\} \Sigma = \{a, b\} \). \( L \) is regular. (TRUE or FALSE?)
(h) \( L = \{a^{2n}b^{3n} \mid n > 0\}, \Sigma = \{a, b\} \). \( L \) is regular. (TRUE or FALSE?)
(i) \( L = \{ w \in \Sigma^* \mid n_a(w) < n_b(w) < 500 \} \) \( \Sigma = \{a, b, c\} \). \( L \) is regular. (TRUE or FALSE?)

(j) \( L = \{a^n b^m c^p \mid p > n, n > 0, m > 0, p > 0\} \). \( L \) is regular. (TRUE or FALSE?)

3. (4 pts) Explain what each of the four parts of 4-tuple for a regular grammar \( G \) represent, \( G = (V, T, S, P) \). Give the formal definition for \( P \).

4. (8 pts) Draw a DFA for the following language. You can show trap states if you want. (show the transition diagram, indicate the start state by a short arrow, and final states by double circles.)

\( L = \{ w \in \Sigma^* \mid w \) has the substring \( aba \) and \( w \) does NOT have the substring \( bb \}\}, \( \Sigma = \{a, b\} \).

For example, \( abaaba, aaaba \) and \( abaaa \) are in \( L \).

5. (6 pts) Write an NFA that is equivalent to the following regular expression: \( a(b + ca)^* \).

6. (6 pts) Write a context-free grammar for the following language.

\( L = \{a^n b^m c^{2n+m} \mid m > 0, n > 0\} \)

7. (6 pts) Consider the following DFA.

a) Show states \( q_0 \) and \( q_3 \) are distinguishable with an appropriate string. Explain.

b) Give the states in the minimal state DFA (you do not need to show the arcs). Each state should indicate which states it represents from the original DFA. For example you could list one state as 0,1,2 if states 0, 1 and 2 in the original DFA can be combined to form a state in the minimal state DFA.
8. (10 pts) Consider \( L = \{a^n b^m c^{2n+m} \mid n \geq 0, m > 0 \} \). Draw the transition diagram for a nondeterministic pushdown automaton \( M \) that accepts \( L \) by final state. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are \( a, b; \ cd \) where \( a \) is the symbol on the tape, \( b \) is the symbol on top of the stack that is popped, and \( cd \) are pushed onto the stack (with \( c \) on top of \( d \)). \( Z \) is on top of the stack when \( M \) starts.)

(a) First list 3 strings in \( L \).

(b) Now draw the transition diagram.

9. (6 pts) **Pumping Lemma:** Let \( L \) be an infinite regular language. \( \exists \) a constant \( m > 0 \) such that any \( w \in L \) with \( |w| \geq m \) can be decomposed into three parts as \( w = xyz \) with

\[
\begin{align*}
|xy| & \leq m \\
|y| & \geq 1 \\
x y^i z & \in L \quad \text{for all } i \geq 0
\end{align*}
\]

**Use the Pumping Lemma to prove** the language \( L \) below is not regular.

\( L = \{ w \in \Sigma^* \mid n_a(w) > n_b(w) + n_c(w) \} \) \( \Sigma = \{a, b, c\} \).

\( \Sigma = \{a, b, c\} \). For example, \( aaaaacb \) and \( abaca \in L \).

**Proof:** (SHOW ALL STEPS! Some have been started for you.)

Assume ________________________________

Choose \( w = ________________________________ \)

10. (8 pts) Consider the following property, DoubleLastA. If \( L \) is a regular language, then \( \text{DoubleLastA}(L) \) = strings from \( L \) that have the last \( a \) replaced with \( aa \). If there is a \( w \) from \( L \) with no \( a \) in the string, then \( w \) is in DoubleLastA unchanged.

For example, if \( caabb \) is in \( L \), then \( caaabb \) is in \( \text{DoubleLastA}(L) \). If \( abbab \) is in \( L \), then \( abbaaab \) is in \( \text{DoubleLastA}(L) \). If \( bb \) is in \( L \), then \( bb \) is in \( \text{DoubleLastA}(L) \).

Prove that \( \text{DoubleLastA}(L) \) is a regular language.