1. (14 pts) Complete or answer the following. If the resulting answer has less than 10 items, just list the items. Otherwise give a formal description of the language.

   \[ L_1 = \{ a, c \}, \Sigma = \{ a, c \} \]
   \[ L_2 = \{ a, bb \}, \Sigma = \{ a, b \} \]
   \[ L_3 = b^* ba^*, \Sigma = \{ a, b \} \]
   \[ L_4 = \{ w \in \Sigma^* | n_a(w) > n_b(w) \}, \Sigma = \{ a, b \} \]

   (a) \( L_1 \times \{ a \} = \)
   (b) \( 2^{|L_1|} = \)
   (c) \( L_1 \cup L_2 = \)
   (d) \( L_1 \cap L_3 = \)
   (e) \( L_3 \cap L_4 = \)
   (f) \( L_3 = \)
   (g) \( L_1 \circ L_3 = \)

2. (20 pts) Answer TRUE or FALSE to each of the statements below.

   (a) \( \emptyset \in \{ \{ \emptyset \}, a, \{ a \}, \{ \emptyset, a, b \} \}? \) (TRUE or FALSE?)
   (b) If there is an NPDA \( M \), then there exists a regular grammar \( G \) such that \( L(M) = L(G) \). (TRUE or FALSE?)
   (c) If \( L \) is regular, then \( L \cap \{ a^n b^n | n > 0 \} \) is regular. (TRUE or FALSE?)
   (d) If \( G \) is a regular grammar then there is not always a unique parse tree for every string \( w \in L(G) \). (TRUE or FALSE?)
   (e) If there is an NFA with only 1 final state, then there exists an equivalent DFA with only 1 final state. (TRUE or FALSE?)
   (f) If \( r \) is a regular expression, then there exists a CFG \( G \) such that \( L(r) = L(G) \). (TRUE or FALSE?)
   (g) \( L = \{ w \in \Sigma^* | n_a(w) = n_b(w) \text{ and } n_b(w) \text{ is even} \} \Sigma = \{ a, b \} \). \( L \) is regular. (TRUE or FALSE?)
   (h) \( L = \{ a^{2n+1} | n > 0 \}, \Sigma = \{ a \} \). \( L \) is regular. (TRUE or FALSE?)
(i) \( L = \{a^n b^{3n} \mid 5000 > n > 0\}, \Sigma = \{a, b\} \). L is regular. (TRUE or FALSE?)

(j) \( L = \{a^n b^m c^p \mid m < n + p, p < 100, n > 0, m > 0, p > 0\} \). L is regular. (TRUE or FALSE?)

3. (4 pts) Explain what each of the parts of a 5-tuple for a deterministic finite automaton represent, \( M = (Q, \Sigma, \delta, q_0, F) \). Give the formal definition for \( \delta \).

4. (8 pts) Draw a DFA for the following language. You can show trap states if you want. (show the transition diagram, indicate the start state by a short arrow, and final states by double circles.)
\( L = \{w \in \Sigma^* \mid w \) has the substring bba and w does NOT have the substring aa\}, \( \Sigma = \{a, b\} \).

For example, bbbab, bba and abba are in L.

5. (6 pts) Write an NFA that is equivalent to the following regular expression: \((bc + a)^*b\).

6. (6 pts) Write a context-free grammar for the following language.
\( L = \{a^{2n+m} b^n c^m \mid m > 0, n > 0\}\)

7. (6 pts) Consider the following DFA.

![DFA Diagram]

a) Show states \( q_0 \) and \( q_3 \) are distinguishable with an appropriate string. Explain.

b) Give the states in the minimal state DFA (you do not need to show the arcs). Each state should indicate which states it represents from the original DFA. For example you could list one state as 0,1,2 if states 0, 1 and 2 in the original DFA can be combined to form a state in the minimal state DFA.
8. (10 pts) Consider $L = \{a^{n+2m}b^m c^n \mid n \geq 0, m > 0 \}$. Draw the transition diagram for a nondeterministic pushdown automaton $M$ that accepts $L$ by final state. (Remember to identify the start state by an arrow and final states by double circles. Format of labels are $a, b; cd$ where $a$ is the symbol on the tape, $b$ is the symbol on top of the stack that is popped, and $cd$ are pushed onto the stack (with $c$ on top of $d$). $Z$ is on top of the stack when $M$ starts. ).

(a) First list 3 strings in $L$.

(b) Now draw the transition diagram.

9. (6 pts) **Pumping Lemma:** Let $L$ be an infinite regular language. $\exists$ a constant $m > 0$ such that any $w \in L$ with $|w| \geq m$ can be decomposed into three parts as $w = xyz$ with

$$
\begin{align*}
|xy| &\leq m \\
|y| &\geq 1 \\
x y^i z &\in L \quad \text{for all } i \geq 0
\end{align*}
$$

**Use the Pumping Lemma to prove** the language $L$ below is not regular.

$L = \{ w \in \Sigma^* \mid n_a(w) > 2 \times n_b(w) + 1 \}$ $\Sigma = \{a, b\}$.

$\Sigma = \{a, b, c\}$. For example, $aaaaba$ and $aaaabaaba \in L$.

**Proof:** (SHOW ALL STEPS! Some have been started for you.)

Assume ____________________________________________________________________________

Choose $w = ____________________________________________________________________________$

10. (8 pts) Consider the following property, ReplaceAReplaceB. If $L$ is a regular language, then ReplaceAReplaceB($L$) is strings from $L$ that have one $a$ and one $b$ removed. If there is a string $w$ that does not have at least one $a$ or one $b$ in the string, then $w$ is NOT in ReplaceAReplaceB.

For example, if $caabb$ is in $L$, then $cab$ is in ReplaceAReplaceB($L$). If $abab$ is in $L$, then $bab$ is in ReplaceAReplaceB($L$) (note it does not matter which $a$ or $b$ is replaced.) If $bb$ is in $L$, then $bb$ is NOT in ReplaceAReplaceB($L$).

Prove that ReplaceAReplaceB($L$) is a regular language.