1. (10 pts) Complete the following.

   (a) \{a\} \times \{a, b\} =
   (b) | \{a, b, c\} \circ \{a, b, c, d\} | =
   (c) 2\{a\} \cup \{a, b\} =
   (d) \{a^n b^m c^n | n > 0, m > 0\} \cap \{a^p b^q c^r | p > 0, q > 0\} =
   (e) Right quotient is defined as \( L_1 / L_2 = \{x | xy \in L_1 \text{ for some } y \in L_2 \} \)

\( L_1 = \{a^n b^m c^n | n > 0, m > 0\}, L_2 = \{b, c\}^* \),

\( L_1 / L_2 = \)

2. (12 pts) Answer TRUE or FALSE to each of the statements below.

   (a) \{\emptyset\} \in \{\emptyset, \{a\}, \{a, b\}\}
   (TRUE or FALSE?)
   (b) \( L = \{b^n a^m | n > m, m > 0, n \text{ is odd } \} \). \( L \) is regular.
   (TRUE or FALSE?)
   (c) \( L = \{b^n a^m | m > 0, n > 0\} \cup \{a^n b^m | m > 0, n > 0\} \). \( L \) is regular.
   (TRUE or FALSE?)
   (d) \( \Sigma = \{a, b\}, L = \{w \in \Sigma^* | n_a(w) \mod 100 = 2\} \). \( L \) is regular.
   (TRUE or FALSE?)
   (e) \( G \) is a regular grammar. There exists a CFG \( G' \) such that \( L(G) = L(G') \).
   (TRUE or FALSE?)
   (f) \( G \) is a CFG with 1 rule. Then there exists a DFA \( M \) such that \( L(G) = L(M) \).
   (TRUE or FALSE?)

3. (3 pts) Give an example of \( L_1 \) and \( L_2 \) such that \( L_1 \) is regular, \( L_2 \) is NOT regular and \( L_1 \circ L_2 \) is regular.

4. (9 pts) Draw a DFA (not an NFA!) for the following language. Do not show trap states.
   (show the transition diagram, indicate the start state by a short arrow, and final states by double circles.)

   \( L = \{w \in \Sigma^* | n_b(w) \text{ is even and } aab \text{ is not a substring of } w\} \), \( \Sigma = \{a, b\} \).

5. (9 pts) Convert the following NFA to a DFA using the algorithm discussed in class.
6. (9 pts) Convert the following DFA to a minimum state DFA. **Show the tree** distinguishing the states and **explain** at each level the reason for distinguishing the states. **Show the resulting minimal DFA** with states labeled with names from their original states (for example, combined states A and B would be called state AB).

![DFA Diagram]

7. (17 pts) Consider the following DFA M.

![DFA Diagram]
(a) (8 pts) Give a regular expression \( R \) such that \( L(R) = L(M) \).
(b) (9 pts) Give a regular grammar \( G \) such that \( L(G) = L(M) \).

8. (9 points) Consider the following language. \( L = \{ a^nb^mc^p \mid p = m + n, n > 0, m \geq 0 \} \)

(a) Write a context-free grammar for \( L \).
(b) Show the parse tree for \( aaabcccc \).

**Pumping Lemma:** Let \( L \) be an infinite regular language. \( \exists \) a constant \( m > 0 \) such that any \( w \in L \) with \( |w| \geq m \) can be decomposed into three parts as \( w = xyz \) with

\[
|xy| \leq m \\
|y| \geq 1 \\
x^iyz \in L \quad \text{for all } i \geq 0
\]

9. (9 pts) Use the Pumping Lemma to prove \( L = \{ a^nb^mc^p \mid p = m + n, n \geq 0, m \geq 0 \} \) is not regular.

**Proof:** (SHOW ALL STEPS! Some have been started for you.)

Assume

Choose \( w = \ldots \)

10. (3 pts) Consider the following proof to prove the language \( \Sigma = \{ a, c \}, L = \{ w \in \Sigma^* \mid n_a(w) = 2 \cdot n_c(w) \} \).

**Proof:**

Assume \( L \) is regular.

Define the homomorphism \( h(L) \) by \( h(aa) = a \) and \( h(c) = b \).

\( L_2 = h(L) = \{ w \in \Sigma^* \mid n_a(w) = n_b(w) \} \), with \( \Sigma = \{ a, b \} \). \( L_2 \) is regular by closure of homomorphism.

\( L_3 = \{ a^n b^n \mid n \geq 0, m \geq 0 \} \), \( L_3 \) is regular.

\( L_4 = L_2 \cap L_3 = \{ a^n b^n \mid n \geq 0 \} \), \( L_4 \) is regular by closure of intersection.

\( L_5 = L_4 - \{ \epsilon \} = \{ a^n b^n \mid n > 0 \} \), \( L_5 \) is regular by closure of difference.

Contradiction! already shown \( L_5 \) is not regular.

Thus, \( L \) is not regular. QED.

Explain what is wrong with the proof.
11. (10 pts) Consider the following property, AllButFirstReplace with \( \Sigma = \{a, b, c\} \).

AllButFirstReplace(L) = \{w \mid w \in L, \text{ and } w \text{ does not contain an } a\} \cup \{wax \mid wav \in L, \text{ w does not contain an } a \text{ and } x \text{ is v with every occurrence of } a \text{ in } v \text{ replaced by } a \ b \} \)

The property AllButFirstReplace applied to a language L accepts strings from L with all but the first occurrence of a replaced with b. For example, if the string babaab is in L, then babbbb is in AllButFirstReplace(L). If aabcab \( \in L \), then abbcab \( \in \) AllButFirstReplace(L). If bcb \( \in L \), then bcb \( \in \) AllButFirstReplace(L).

Prove that the regular languages are closed under the AllButFirstReplace(L) property. (Show all steps!)